Feedback linearization control for a nonlinear chaotic system using differential evolution algorithm

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ABSTRACT

In this paper, a tracking problem for a nonlinear chaotic system is investigated. Firstly, we propose a new optimal estimation approach, which is based on a differential evolution (DE) to parameters identification, for the system. According to the estimated result, the techniques of sliding mode control and feedback linearization are addressed to control the nonlinear chaotic system track the desired signal.

KEY WORDS

Differential evolution, System identification, Genesio-Tesi Chaotic system, Sliding mode control

INTRODUCTION

System identification is a quite important and essential work for many engineering applications such as control system engineering. When a mathematical dynamic expression for an actual system is derived, then a variety of controllers can be designed by using this mathematical model to achieve certain control performance. In the last few years, many articles have been shown that the artificial intelligence approach is successfully applied to solving the system identification problem, for example, using neural networks [1] and fuzzy systems [2].

In this paper, we propose an optimal differential evolution (DE) algorithm to identify the system parameters of a nonlinear chaotic system. The DE algorithm was the first proposed by Storn and Price [3]. The good feature of a DE approach is that its algorithm is easy to understand and run, because it is performed based on using a real-valued representation directly, not necessary to be transferred as the form of binary strings. Another advantage of the DE algorithm is that it needs to evaluate the cost function alone to guide its search. There is no requirement for derivative that is often requested in solving for traditional optimization problems, e.g. the gradient method.

The estimated system model is then applied to the feedback linearization control [4] such that the system can be transferred to a linear model with a nonlinear bounded time-varying uncertainty. To deal with the uncertainty, a sliding mode control approach is designed to achieve the tracking control.

MAIN RESULT

We consider a class of nth-order nonlinear system, with the input $u \in \mathbb{R}$ and the output $y \in \mathbb{R}$, given by

$$
\begin{align*}
\dot{x}_1 &= x_2, \quad \dot{x}_2 = x_3, \ldots, \quad \dot{x}_{n-1} = x_n, \\
\dot{x}_n &= f(X, \Theta) + u, \\
y &= x_1,
\end{align*}
$$

where $X = [x_1, x_2, \ldots, x_n] \in \mathbb{R}^n$ and $\Theta = [\theta_1, \theta_2, \ldots, \theta_n]$
are the state and parameter vectors of the system, respectively, and \( f(\cdot) \) is the nonlinear function. However, under many physical situations, the exact parameter \( \Theta \) can not be easily obtained. Therefore, we need to develop an estimated nonlinear model, which is expected to match the actual system of (1).

Based on the a differential evolution (DE) to parameters identification for nonlinear systems (1), by using the technique of feedback linearization [4], the system (1) can be transferred to a linear model with a nonlinear bounded time-varying uncertainty

\[
\dot{x}_1 = x_2, \\
\dot{x}_2 = x_3, \\
\vdots \\
\dot{x}_{n-1} = x_n, \\
\dot{x}_n = v + \Delta,
\]

where \( \Delta \) represents the error of estimated. Since less error can be achieved by using the a DE to parameters identification, the uncertain \( \Delta \) is bounded.

Furthermore, define the tracking error as \( e_i = y - y_d \) and let

\[
e_i = x_i - y_d^{(i)}, \\
\hat{e}_i = \hat{x}_i - y_d^{(i)}, \]

The state equations for \( e_i \) to \( e_n \) are

\[
\dot{e}_i = e_{i+1}, \\
\hat{e}_i = \hat{e}_{i+1} = e_{i+1} - v + \Delta - y_d^{(i+1)}.
\]

Now, our objective is to design the sliding mode control to ensure that \( e = [e_1 \cdots e_n]^T \) is bounded and \( e \to 0 \) as \( t \to \infty \). Thus, we have the following theorem.

**Theorem 1:** If we choose a sliding surface

\[
\sigma = k_1 e_1 + k_2 e_2 + \cdots + k_n e_n + e_n,
\]

where \( k_1 \) to \( k_n \) are chosen such that \( \sigma = k_{n+1} e_{n+1} + \cdots + k_1 \) is Hurwitz, then the sliding control law

\[
v = -k_1 \hat{e}_1 - k_2 \hat{e}_2 - \cdots - k_n \hat{e}_n + y_d^{(n)} - k \text{sgn}(\sigma)
\]

where \( k > |\lambda| \), will be such that \( e = [e_1 \cdots e_n]^T \) is bounded and \( e \to 0 \) as \( t \to \infty \).

**AN ILLUSTRATIVE EXAMPLE**

An illustrative example of interest is a nonlinear Genesio-Tesi chaotic system, and its dynamic differential equations are described by [5]

\[
\dot{x}_1 = x_2, \\
\dot{x}_2 = x_3, \\
\vdots \\
\dot{x}_{n-1} = x_n, \\
\dot{x}_n = -\theta_1 x_1 - \theta_2 x_2 - \theta_3 x_3 + x_1^3 + u, \\
y = x_1,
\]

where \( \theta_1 = 6, \theta_2 = 2.92, \theta_3 = 1.2 \). We assume that the sampling time is equal to 0.001 for simulating the differential chaotic dynamic equations, and that the initial system states are given by \( x_1(0) = 1, x_2(0) = 1, \) and \( x_i(0) = 1 \), respectively. The estimated parameters have entered the steady state after about performing 200 generations, and eventually converge to the actual system parameters accurately, i.e., \( \theta_1 = 6, \theta_2 = 2.92, \) and \( \theta_3 = 1.2 \), respectively. According to Theorem 1, we choose \( k = 5, k_1 = 4 \), and \( k_2 = 4 \), output \( y \) will track the desired output \( y_d = \cos(t) \).

**CONCLUSION**

In this paper, the differential evolution algorithms have been successfully applied to solve the parameters estimation for a class of nonlinear systems. Based on it, we use the technique of feedback linearization and sliding mode control approach to design controller for tracking problem. Simulation result for a Genesio-Tesi chaotic system is made to verify the validity of the proposed scheme.

**REFERENCES**