Combining Digital Pens with Interactive Support Systems for Learning Mathematics

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Abstract

The paper-based textbook is the main medium for learning mathematics in schools because it has many advantages including portability, intuitive annotation, and ease of navigation. However, students with lower reading comprehension ability cannot easily solve mathematical word problems in paper, and this impedes their learning and motivation. This study thus proposes a paper-student-computer interaction model, in which computers monitor the learning behavior of students working on paper-based textbook through digital pens and thereby provide interactive learning support. An experiment was conducted to determine the extent to which the proposed approach facilitated mathematical learning of seven junior high school students. The results revealed that this model is helpful in reducing their cognitive load when reading complex and interleaved mathematical word problems in paper-based textbook. This effect can therefore indeed facilitate the mathematical learning performance of these students.

Keyword: Computer support learning system, Digital pens, Mathematical word problem

1. Introduction

Mathematics ability is a fundamental and indispensable competence in K-12 education. The paper-based textbook is the main medium for learning mathematics in schools because it can afford many advantages such as portability, intuitive annotation, and easy of navigation [1]. Students use pens to freely write the solutions of mathematical word problems. However, some students have difficulty solving mathematical word problems. Of these difficulties, reading comprehension and problem solving behavior need to be addressed specially as some students often leads to lower reading comprehension ability and intuitive problem solving behavior.

In regard to reading comprehension, some students tend to focus only on fragments of text or single words [2]. Therefore, it is difficult for these students to integrate separate pieces of information to obtain a contextual understanding of the mathematical word problem as a whole [3]. Regarding problem solving behavior, some students tend to generate intuitive solutions quickly and make errors easily when solving a mathematical word problem, due to the lack of contextual understanding of the problem. Such intuitive behaviors may impede the self-reflection and critical thinking which are essential in advancing their ability to solve new problems [4].

Educators have indicated that excessive cognitive load impedes the learning and motivation of students [5]. Because some students experience a heavy cognitive load when reading and solving mathematical word problems in paper, an advanced support system that can help them understand and solve the problems in this environment is clearly indicated. An ever increasing array of digital pen and paper technology is being developed which can bridge the paper and digital worlds [6]. This digital pen and paper technology could be used to help learners learn through augmenting autonomous paper-based activities with the aid of computers. By capturing the learner’s paper activity using a digital pen, what the learner does on paper can be conveyed to and processed by the computer. With the interactive features of digital pen and paper technology, learning can be facilitated through computer mediated contextualized guidance based directly upon learning activity taking place on paper.

Based on the features of digital pen and paper technology, this study proposes a new approach to aid students working on paper by providing interactive
learning support through the use of digital pen input. More specifically, the computer monitors students’ learning behaviors on paper and provides contextualized guidance to help them solve mathematical word problems. It is hoped that the interactive learning support enabled by the digital pens can reduce the cognitive load of students and thereby enhance their problem understanding and solving capabilities. Working with seven hard of hearing students from a self-contained class who are solving mathematical word problems with this approach, this study attempts to gain a better understanding of a framework that could describe how best to facilitate these students to learn mathematics in paper.

2. Related Work

When solving mathematical word problems, some students do not easily develop a mental representation of problems, due to lower reading comprehension ability, and consequently they often resort to solving problems intuitively. Therefore, the main difficulty these students have with mathematical word problems is building appropriate problem representation with which to generate solutions. To address this issue, Mousley and Kelly (1998) adopted graphics, which depict the context of mathematical word problems, to help students understand the mathematical word problems graphically, as graphics can make visible those mathematical relationships and constraints that are not easily seen [7]. The results of the study suggest that graphics can be used to help students build problem representation to solve mathematical word problems.

Although graphics can help students understand the context of mathematical word problems, the study [8] further indicated that static graphics cannot help students solve mathematical word problems effectively if the mathematical word problems involve dynamic changes of state. This is because static graphics cannot clearly express complicated states in mathematical word problems. They suggested that it may be necessary to use dynamic graphics to facilitate students in understanding sequences of states in mathematical word problems. In other words, a support system is needed which helps students not only to understand the static states in mathematical word problems, but also to guide them interactively through the solution process [7]. However, such a support system is not easy to implement in a paper-based learning environment due to the limitations of paper in providing dynamic graphics and interactive support. It is therefore necessary to find an approach that can integrate paper and computers to support students in solving mathematical word problems.

Computers have been used to provide interactive support for students to help them learn mathematics. In particular, computers were applied to help students understand the semantic structure of mathematical word problems. For example, the study [9] utilized a computer game to represent a mathematical word problem as part-part-whole relationships to help students understand the sequences of states involved in the problem. Moreover, computers have been applied to facilitate students in solving mathematical word problems. The website “Project Solve” provides mathematical word problems which students can practice on the Internet [10]. Computers were applied to provide interactive instruction and guided practice to assist students in solving mathematical word problems. Such an approach can enhance problem solving performance because the interactive instruction and guided practice facilitate students in understanding and building mental representations of the problems.

In recent years, Liu et al. (2006) proposed a methodology to help students solve mathematical problems on Tablet PCs [11]. Their study indicated that decomposing complex problems into sub-problems and providing interactive guidance for solving these sub-problems could help students solve the complex problems. This was the result of reducing students’ cognitive load and prompting them to think carefully about the complex problems, rather than solving the problems impulsively.

The aforementioned studies suggest that, when guiding students in solving mathematical word problems, it is critical to increase problem understanding, reduce cognitive load, and provide interactive instructions. These studies have confirmed that computers can help students solve mathematical word problems by providing dynamic representations of those problems and can prompt students with interactive learning activities. However, it is difficult to provide such learning supports for students to solve mathematical word problems in paper, since hardcopy can only provide static content. Therefore, it is necessary to integrate paper with computers in such a way that the computer can provide the dynamic and interactive learning support in direct response to the learning activity taking place on paper. In other words, as students solve mathematical word problems in paper, the computer must monitor their learning behaviors on paper and thereby provide responsive learning support.

Another approach to enhancing the interactivity between the computer and learners working on paper is digital pen and paper technology. The learners work as if they were working on a traditional paper medium, but do so with digital pens which are equipped with monitoring capability whereby the pens sense and capture pen strokes. The digital pen can possibly enhance learning by integrating paper learning materials with computers. For example, digital pens have been applied to transforming individual annotation activity into a social learning style. In this study [6], learners used digital pens to take notes on paper. Because the pens were able to sense the pen strokes, notes could be shared among learners through a computer.
3. Methods

This study uses a digital pen which includes both a pen and an ultrasonic signal receiver in the implementation of the interactive learning support system illustrated in Figure 1. The digital pen is an ultrasonic transmitter which emits signals indicating three pen events: Pen Down, Pen Up, and Pen Move as a student performs these actions. The ultrasonic signal receiver then detects the signals and calculates the coordinates of the location where the event takes place on the paper. The signals are transmitted in real-time to a computer. In other words, the receiver will send a sequence of pen events that include coordinates, timestamps, and pen-event types to the computer. A behavior recognizer in the computer then processes the pen event sequence to identify the learning behavior on paper that the student exhibits.

![Figure 1 The paper-student-computer interaction model](image)

The learning behavior recognizer identifies three types of learning behaviors on paper, including pointing, single clicking, and writing behaviors based on the pen event sequence. The three types of behaviors are addressed because these three represent different important learning states during which students may need different types of support. For example, the pointing behavior, which reflects the fact that the student points to a location on the paper with the pen for a period of time, may indicate that he/she may be distracted or confused by the material at that location. It is then possible to provide him/her with content-specific interactive learning support. Similarly, a student may click a ‘Help’ tag next to the mathematical word problem to request immediate context appropriate help from the computer. On the other hand, if he/she is writing, the system logs his/her pen strokes as an important entry in his/her learning portfolio. Because the three types of behaviors differ in their respective patterns of pen event sequence, the learning behavior recognizer can use the event sequence to determine the type of learning behavior that is taking place. These recognizing rules and corresponding actions reflecting the three learning behaviors on paper are detailed in Table 1.

<table>
<thead>
<tr>
<th>Behavior</th>
<th>Recognizing Rule</th>
<th>System Action</th>
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<tbody>
<tr>
<td>Pointing</td>
<td>Detecting the first pen down event $e_1(x_1, y_1), t_1$, Pen Down &gt; and the first pen up event $e_2(x_2, y_2), t_2$, Pen Up &gt; after $e_1$ satisfying the condition: $t_2-t_1$ &gt; threshold, and $</td>
<td>x_2-x_1</td>
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<tr>
<td>Single clicking (requesting help)</td>
<td>Writing</td>
<td></td>
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<td>----------------------------------</td>
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<td></td>
</tr>
<tr>
<td>Detecting the first pen down event $e_1 &lt; (x_1, y_1), t_1, \text{Pen Down} &gt;$ and the first pen up event $e_2 &lt; (x_2, y_2), t_2, \text{Pen Up} &gt;$ after $e_1$ satisfying the condition: $t_2 - t_1 &lt; \text{threshold}$, and $</td>
<td>x_2 - x_1</td>
<td>&lt; \text{threshold}$, and $</td>
</tr>
<tr>
<td>Saving the pen stroke into the portfolio database</td>
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</table>

Figure 2  Contextualized graphics for a mathematical word problem

The paper-student-computer interaction model can provide interactive learning support which can aid students in solving mathematical word problems based on their learning behaviors identified by the learning behavior recognizer. As mentioned in the previous section, the interactive learning support can reduce the cognitive load that results from the complex interleaved semantic relationships that occur in mathematical word problems. This study thus adopts a decomposition approach to reduce the cognitive load of students in accordance with the taxonomy of mathematical word problems. The taxonomy includes four basic problem types: change problems, combine problems, compare problems, and equalization problems [12]. Based on this taxonomy, the computer, upon having detected that a learning disorder student was pointing to a mathematical word problem or single clicking a ‘Help’ tag on the paper-based textbook, will display a sequence of basic problems decomposed from the mathematical word problem. When the student had seen the first basic problem on the screen of computer, he/she could write the solution on the paper-based textbook. In the meantime, he/she could click the ‘Next’ tag on the paper-based textbook to see and solve the next basic problem on the screen of computer. Since the student only solved one basic problem at a time, this approach has the potential to reduce the cognitive load of the student.

To accommodate the difficulties students encounter in understanding the semantics of mathematical word problems, this study provides contextualized graphics which illustrate the meaning of each mathematical word problem. The contextualized graphics are displayed on the computer to enhance students’ understanding of a mathematical word problem by enabling them to visualize the semantic relationships and constraints involved in the mathematical word problem. Each contextualized graphic incorporates the context of each basic problem and exhibit explicitly the meaning of each basic problem being studied in accordance with its taxonomy. An example that utilizes the decomposition of a mathematical word problem and contextualized graphics to help the student solve the mathematical word problem is shown in Figure 2. In this example, a complex mathematical word problem is decomposed into one combine (Q1), one change (Q3), and two equalization (Q2 and Q4) basic problems. Each graphic generation schema is used to illustrate the meaning of each basic problem. By guiding students with such graphics to solve each basic problem one after another, it is hoped that this approach can reduce the cognitive load of students in solving mathematical word problems.
Furthermore, each student had to take a questionnaire based on Technology Acceptance Model for investigating students’ perceived viewpoints toward the interactive support system. For each statement on the questionnaire, the students indicated their level of agreement using the following five-point Likert scale: 1 – strongly disagree; 2 – disagree; 3 – neither agree nor disagree; 4 – agree; 5 – strongly agree. The statements included on the questionnaire were:

- Question 1: I enjoyed working with the interactive support system.
- Question 2: I easily understood the sequence of basic problems decomposed from the complex mathematical word problem.
- Question 3: I was more efficient in solving the complex mathematical word problems.

4. Experiments and discussions

An experiment was conducted to investigate the extent to which the paper-student-computer interaction model can help students solve mathematical word problems in paper-based textbooks. It was conducted in a self-contained class containing seven junior high school students. These students were to solve five mathematical word problems in fifty minutes in two different settings: the traditional paper and paper-student-computer interaction scenarios. The five mathematical word problems were similar in both of these settings. The gap between the two problem solving activities was two weeks. Each mathematical word problem was previously decomposed by the authors and contained 2-4 basic problems from the aforementioned taxonomy of mathematical word problems. Of these five mathematical word problems, four were change problems, three were combine problems, five were compare problems, and four were equalization problems.

In order to obtain a better understanding of student performance in solving mathematical word problems, after students had finished both of the problem solving activities, the teacher evaluated the problem solving processes, determining whether they had solved each mathematical word problem and each basic problem. Student performance in the two settings was analyzed with Chi-square analysis because student performance was described as category data (correct or incorrect).

Student performance in solving these five mathematical word problems is shown in Table 2. The result indicates that only three students solved three mathematical word problems correctly when working with the traditional paper setting, a correctness count of only 8.57% (3/35). On the other hand, the students, when working with the paper-student-computer interaction setting, were able to solve more mathematical word problems correctly -- five students having solved eleven mathematical word problems correctly. This number indicates a rate of correctness of 31.43% (11/35) which is significantly higher than that achieved in the traditional paper setting ($\chi^2=5.71$; d.f.=1; $p=0.02$). This result reveals that the paper-student-computer interaction model was indeed able to help students improve their performance in solving these mathematical word problems. This result echoes the findings of the study [11], which addressed the question of whether the decomposition of complex mathematical problems with clear instructions for each sub-problem was critical in improving the learning of students. On the other hand, some students still encountered difficulties with solving these mathematical word problems even when facilitated by interactive learning support and contextualized graphics.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Students’ performances in solving mathematical word problems in two settings</th>
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<tr>
<td></td>
<td>% of correctness in a traditional setting</td>
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<tr>
<td>Correctness %</td>
<td>8.57 (3/35)</td>
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<tr>
<th>Table 3</th>
<th>Students’ performances for four basic problem types in two settings</th>
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<tr>
<td></td>
<td>% of correctness in a traditional setting</td>
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<tr>
<td>Change</td>
<td>50 (14/28)</td>
</tr>
<tr>
<td>Combine</td>
<td>23.81 (5/21)</td>
</tr>
<tr>
<td>Compare</td>
<td>25.71 (9/35)</td>
</tr>
<tr>
<td>Equalization</td>
<td>17.86 (5/28)</td>
</tr>
</tbody>
</table>

Student performance of different types of basic problems was further analyzed to gain a clear understanding of the effect of contextualized graphics. The results are shown in Table 3, and reveal that students achieved better performance in the paper-student-computer interaction setting than in the traditional paper setting. In particular, in the paper-student-computer interaction setting, student performance in combine problems was 85.71%, which is significantly higher than that achieved in the traditional paper setting ($\chi^2=16.24$; d.f.=1; $p<0.01$). This improvement in performance was also reflected in the equalization problems. Students solving equalization problems in the paper-student-computer interaction setting.
students have learned more in this experiment. This is confirmed that the support system and solved the complex mathematical problems. Digital pens, which can help students learn mathematics in paper-based textbooks. However, this study reflects only a small-scale investigation. Further work needs to be undertaken with larger sample groups to provide additional evidence. In addition, the experiments were conducted in this specific setting using paper-based textbooks and digital pens as proposed in this study, but the conclusions detailed in the previous paragraph should also be valid in settings using e-book readers in general. iPad and Kindle, for example, provide standard annotation, pen stroke monitoring functionalities, and the sequence of basic problems that would serve the same purpose as the interactive support system. It would be interesting to see the results of a study using commercially available e-book readers.

5. Conclusions

This study explores how digital pens combined with computer support learning systems can help students reduce cognitive load while solving mathematical word problems in paper-based textbooks. A paper-student-computer interaction model to monitor learning behavior on paper through the use of digital pens and provide interactive guidance based on that student learning behavior was proposed to facilitate students in solving mathematical word problems. The results of this study reveal that such a model can help students learn mathematics because it can guide them through the solution of each basic problem with the use of contextualized graphics in computers. It was found that the decomposition of mathematical word problems, together with clear contextualized graphics in solving those decomposed problems does help students work with complex and interleaved mathematical words and expressions on paper. Given the effect that digital pen and paper technology can have, educators may well be able to apply it to the enhancement of learning in other scenarios as well.

The results of this study demonstrate that the decomposition of mathematical word problems and contextualized graphics can help students learn mathematics in paper-based textbooks. However, this study reflects only a small-scale investigation. Further work needs to be undertaken with larger sample groups to provide additional evidence. In addition, the experiments were conducted in this specific setting using paper-based textbooks and digital pens as proposed in this study, but the conclusions detailed in the previous paragraph should also be valid in settings using e-book readers in general. iPad and Kindle, for example, provide standard annotation, pen stroke monitoring functionalities, and the sequence of basic problems that would serve the same purpose as the interactive support system. It would be interesting to see the results of a study using commercially available e-book readers.

6. References


摘要

因為紙本教科書有很多好處包含易攜帶、直覺式註解及易瀏覽，所以是學生在學校中學習數學的主要媒介。然而，低理解能力的學生無法輕易解決在紙本上的數學文字題，因此會阻礙他們的學習與動機。此研究因此提出一個紙本-學生-電腦的模型，電腦能經由數位筆來監控學生在紙本教科書上的學習行為，然後提供立即的互動學習支援。此實驗是經由七位國中生來驗證我們所提出的方法。實驗結果顯示當這些學生在紙本教科書上讀錯綜複雜的數學文字題時，此模型對降低學生的認知負荷是有幫助的，因此此實驗確實能促進這些國中生的數學學習成效。

關鍵字：電腦支援學習系統，數位筆，數學文字題

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