Applications of Particle Swarm Optimization in Mechanical Design

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Abstract

Particle swarm optimization (PSO) has shown to be an efficient, robust and simple optimization algorithm. Most of the PSO studies are empirical, with only a few theoretical analyses that concentrate on understanding particle trajectories. This paper overviews current theoretical studies, and extend these studies to applications in mechatronic systems, such as identification, control gains and optimization in design. Experimental results are provided to support the conclusions drawn from the theoretical findings.

Keyword: Particle swarm optimization (PSO), mechatronic systems, identification, control gains, optimization

I. Introduction

Particle swarm optimization is a stochastic population based optimization approach, first published by Kennedy and Eberhart in 1995 [1,2]. Since its first publication, a large body of research has been done to study the performance of PSO, and to improve its performance. From these studies, much effort has been invested to obtain a better understanding of the convergence properties of PSO. These studies concentrated mostly on a better understanding of the basic PSO control parameters, namely the acceleration coefficients, inertia weight, velocity clamping, and swarm size [3-5]. From these empirical studies it can be concluded that the PSO is sensitive to control parameter choices, specifically the inertia weight, acceleration coefficients and velocity clamping. Wrong initialization of these parameters may lead to divergent or cyclic behaviour.

Among existing evolutionary algorithms, the best-known branch is the real-coded genetic algorithm (RGA). RGA is a stochastic search procedure based on the mechanics of natural selection, genetics and evolution. Compared with RGA, PSO has some attractive characteristics. It has memory, so knowledge of good solutions is retained by all the particles; whereas in RGA, previous knowledge of the problem is discarded once
the population changes. It has constructive cooperation between particles; that is, particle in the swarm share information among themselves.

To date, PSO has been successfully applied to optimizing various continuous nonlinear functions in practice. Such as, we using the PSO algorithm, a novel design method for the self-tuning PID control in a slider-crank mechanism system [6-10], and we propose an effective method for singularity control of a fully parallel robot manipulator using the particle swarm optimization (PSO) and grey prediction (GP) [11-16]. In system identification, we adopt the PSO to find the parameters of the Scott-Russell (SR) mechanism and the piezoelectric actuator (PA) [17-24].

II. Applications

In 1995, Kennedy and Eberhart [1-2] first introduced the particle swarm optimization (PSO) method, which is derived from the social-psychological theory, and has been found to be robust in complex systems. Instead of using evolutionary operators to manipulate the individual particle each particle is treated as a valueless particle in g-dimensional search space, and keeps track of its coordinates in the problem space associated with the best solution (evaluating value) [25-28]. This value is called pbest. Another best value that is tracked by the global version of the particle swarm optimizer is the overall best value, and its location, obtained so far by any particle in the group, this is called gbest.

The PSO concept consists of changing the velocity of each particle toward its pbest and gbest locations. For example [27-28], the jth particle is represented as \( x_j = (x_{j,1}, x_{j,2}, \ldots, x_{j,g}) \) in the g-dimensional space. The best previous position of the jth particle is recorded and represented as \( \text{pbest}_j = (\text{pbest}_{j,1}, \text{pbest}_{j,2}, \ldots, \text{pbest}_{j,g}) \). The index of best particle among all particles in the group is represented by the \( \text{gbest}_g \). The rate of the position change (velocity) for particle \( j \) is represented as \( v_j = (v_{j,1}, v_{j,2}, \ldots, v_{j,g}) \). The modified velocity and position of each particle can be calculated using the current velocity and distance from \( \text{pbest}_{j,g} \) to \( \text{gbest}_{j,g} \) as shown in the following formulas [28-29]:

\[
\begin{align*}
    v_{j,g}^{(t+1)} &= w \cdot v_{j,g}^{(t)} + c_1 \cdot \text{rand}(t) \cdot (\text{pbest}_{j,g} - x_{j,g}^{(t)}) + c_2 \cdot \text{Rand}(t) \cdot (\text{gbest}_g - x_{j,g}^{(t)}), \\
    x_{j,g}^{(t+1)} &= x_{j,g}^{(t)} + v_{j,g}^{(t)}.
\end{align*}
\]

where \( j = 1, 2, \ldots, n; \quad g=1, 2, \ldots, m \)

where \( n \) is the number of particles in a group; \( m \) is the number of members in a particle; \( t \) is the pointer of iterations (generations); \( v_{j,g}^{(t)} \) is the velocity of the particle \( j \) at
iteration \( t \), \( V_{g}^{\min} \leq v_{j,g}^{(t)} \leq V_{g}^{\max} \); \( w \) is the inertia weight factor; \( c_1 \), \( c_2 \) is the acceleration constant; \( \text{rand()} \), \( \text{Rand()} \) is random number between 0 and 1; \( x_{j,g}^{(t)} \) is the current position of particle \( j \) at iteration \( t \); \( p_{\text{best}j} \) is the pbest of particle \( j \); \( g_{\text{best}g} \) is the gbest of the group \( g \).

In the above procedures, the parameter \( V_{g}^{\max} \) determined the resolution, or fitness, with which regions were searched between the present position and the target position. If \( V_{g}^{\max} \) is too high, particles might fly past good solutions. If \( V_{g}^{\max} \) is too low, particles may not explore sufficiently beyond local solutions.

The constants \( c_1 \) and \( c_2 \) represent the weighting of the stochastic acceleration terms that pull each particle toward pbest and gbest positions. Low values allow particles to roam far from the target regions before being tugged back. On the other hand, high values result in abrupt movement toward, or past, target regions.

Suitable selection of inertia weight \( w \) provides a balance between global and local explorations, thus requiring less iteration on average to find a sufficiently optimal solution. As originally developed, \( w \) often decreases linearly from about 0.9 to 0.4 during a run. In general, the inertia weight \( w \) is set according to the following equation \([9]\):

\[
w = w_{\max} - \frac{w_{\max} - w_{\min}}{\text{iter}_{\max}} \times \text{iter}
\]

where \( \text{iter}_{\max} \) is the maximum number of iterations (generations), and \( \text{iter} \) is the current number of iterations.

### II.1 Parameters Identification

In recent years, the piezoelectric actuator (PA) is more and more widely used in modern precision and driving systems, the Scott-Russell (SR) amplifying mechanism is designed for a cutting tool to amplify the PA displacement and carries out high precision for the motion control system. To analyze the hysteresis phenomenon of the PA, there are many researches concerning about modeling and identifying the hysteresis behavior for the purpose of solving non-linear problems. Therefore, mathematical modeling of the physical systems with hysteresis phenomena is an important tool. In this section of this study is to utilize Bouc-Wen hysteresis models, to identify a PA, which drives a Scott-Russell magnification mechanism. In system identification, we adopt the PSO to find the parameters of the SR mechanism and the PA.
II.1.1 Bouc-Wen Hysteresis Models

Bouc-Wen Model [30] is a typical hysteretic nonlinear model, which was originally proposed by Bouc in 1967 and later generalized by Wen in 1976. This model is described by a first-order nonlinear differential equation as follows:

\[ \dot{h} = \alpha \dot{V} - \beta |\dot{h}|^{n-1} - \gamma |h|^n \]  

(3)

where \( h \) represents the hysteretic state variable and \( V \) is the input voltage. The parameter \( \alpha \) controls the hysteretic amplitude, \( \beta \) and \( \gamma \) control the shapes of hysteresis loop, and \( n \) controls the smoothness of transition from elastic to plastic responses. The identified model has not only been used to forecast the behavior of hysteretic elements but also design controllers for nonlinear systems.

The schematic diagram of the SR amplifying mechanism actuated by a PA, and its dynamic equation is established as follows [31-32]:

\[ m\ddot{x} + c\dot{x} + kx = k(d_e - h) \]  

(4)

where \( m \) is the equivalent mass, \( c \) is the equivalent damping, \( k \) is the equivalent stiffness, \( x \) is the output displacement at point \( D \) of the SR amplifying mechanism, and \( d_e \) is the piezoelectric coefficient.

II.1.2 The PSO Method

The PSO is an optimization searching algorithm, which simulates evolution mechanism on a computer-based platform in conjunction with natural selection and genetic mechanism. The chromosomes are expressed by vectors and each element of vectors is called pbest. The initial pbest in the chromosomes are gotten through generating a sequence by a randomly limited range. All chromosomes compose of a population and are evaluated according to the pre-given evaluation index and given fitness values. According to the fitness values, the reproduction, crossover and mutation operations are carried out among or between the chromosomes. The PSO continuously searches the better chromosomes in this way until the converging index is satisfied.

How to define the fitness function is the key point of the PSO, since the fitness function is a figure of merit, and could be computed by using any domain knowledge. In this section, we adopt the fitness function as follows [33]:

\[ F(\text{parameters}) = \sum_{i=1}^{n} E_i^2 \]  

(5a)

\[ E_i = x_i^{(i)} - x_i^{*(i)} \]  

(5b)

where \( n \) is the total number of samples and \( E_i \) is the calculated error of the \( i \)th sampling time, \( x_i^{*(i)} \) is a solution by using the fourth-order Runge-Kutta method to
solve the dynamic equation (4) of the SR amplifying mechanism with the parameters identified from the PSO method, and \( x^{(i)} \) is the displacement measured experimentally at the \( i \)th sampling time.

II.2 System Control Gains

During the past decades, process control techniques in the industry have made great advances. Among these the best known is the proportional integral derivative (PID) controller. Over the years, several heuristic methods have been proposed for the tuning of PID controllers. PSO is one of the modern heuristic algorithms. Because the PSO method is an excellent optimization methodology and a promising approach for solving the self-tuning PID controller parameters problem, this controller is called the PSO self-tuning PID controller. By using the PSO algorithm, a novel design method for the self-tuning PID control in a slider-crank mechanism system. By using the PSO approach, both the initial PID parameters under normal operating conditions and the optimal parameters of PID control under fully-loaded conditions can be determined. The proposed self-tuning PID controller will automatically tune its parameters within these ranges.

II.2.1 The PSO Algorithm

The PSO algorithm [34] was mainly utilized to determine three optimal PID controller parameters: \( kp, ki, \) and \( kd \). We use the “individual” to replace the “particle” and the “population” to replace the “group” in this section. We defined three controller parameters to compose an individual \( K \) by \( K := [kp, ki, kd] \) with its dimension being \( n \times 3 \). A set of good control parameters \( kp, ki, \) and \( kd \) can yield a good step response and result in minimization of performance criteria in the time domain. These performance criteria in the time domain include the settling time \( t_s \), rising time \( t_r \), maximum overshoot \( M_p \), and steady-state error \( e_{ss} \). In the meantime, we define the evaluation function \( f_1 \), which is a reciprocal of the performance criterion, as [6]:

\[
 f_1 = \frac{1}{(1-e^{-\beta}) \cdot (M_p + e_{ss}) + e^{-\beta} \cdot (t_s-t_r)}. 
\]  

where \( \beta \) is the weighting factor.

II.2.2 Self-Tuning PID Controller

The most significant of the proposed PID tuning method is that the initial parameters and tuning ranges of PID controllers are supported by the PSO approach. It will increase confidence in the controller for operators. After this, the tuning work will be completed by the fuzzy rule. In order to limit the \( kp, ki, \) and \( kd \) within a reasonable
range, Routh-Hurwitz criterion must be employed to test the stability of the closed-loop system. If the $kp$, $ki$, and $kd$ satisfy Routh-Hurwitz stability test, which is applied to the characteristic equation of the system, then the self-tuning PID controller is stable.

II.3 Optimization Design

We propose an effective method for singularity control of a fully parallel robot manipulator using the PSO and grey prediction (GP). We demonstrated in detail how to employ the PSO and GP methods to search efficiently the optimal damping values on-line, the results automatically adjust the damping value around the singular point, and greatly improve the accuracy of system responses. In this section, the PSO method is employed to find seven optimal data for constructing the GP model, which is adapted to obtain the optimal damping values, and effectively damp the end-effector accelerations when the manipulator is in the neighborhoods of singularities. The control using both damped acceleration and damped velocity is called the hybrid damped resolved-acceleration control scheme (HDRAC).

In this section, the HDRAC scheme is used to solve for the damping factors $\rho$ by the PSO and GP optimization methods. Damping factors $\rho$ are not fixed values, and are used to damp accelerations of the point $P$, when the manipulator is in the neighborhood of singularities. For on-line applications, the optimization method can immediately provide optimal damping factors.

II.3.1 Evaluation Function Definition

The PSO algorithm is mainly utilized to determine the optimal damping value $\rho$. In the meantime, we define the evaluation function $f_2$:

$$f_2 = \frac{\sigma}{\sigma^2 + \rho^2} a_d$$

where we assume $\|a_d\| = 1$, and specify $\|f_2\| \leq 25$. The singular region is defined as $\sigma < 0.1$. $\sigma$ is the smallest singular value, singular parameter, or the manipulability measurement.

III. Case studies

III.1 Parameters Identifications of a Scott-Russell

III.1.1 The Scott-Russell Mechanism

The rigid-body diagram of the SR amplifying mechanism is shown in Fig. 1, and it is composed of two links AC and BD, where a pin at point A connects these two links,
point C is a fixed point, and the lengths of AB, AC, and AD are the same.

Fig. 1. The schematic diagram of the SR amplifying mechanism.

The piezoelectric actuator is excited by an applied voltage and acts at point B against the Y axis. Then, through the SR amplifying mechanism, the motion is transferred to point D in the X axis. The PA will elongate or shorten due to the applied voltage being positive or negative value and the link AC rotates and generates linear motion at point D.

The displacement analysis of the SR amplifying mechanism is also shown in Fig. 2, where $\theta$ is the initial angle, $\Delta \theta$ is the angular displacement due to an input incremental displacement $\Delta y_B$ at point B, and $\Delta x_D$ is the output incremental displacement at point D.

Fig. 2. The rigid-body diagram of the SR amplifying mechanism.

The input Y-axis motion of point B is transferred to the output X-axis motion at point D.
III.1.2 Experimental setup

The configured experimental setup is shown in Fig. 3 (a), while the photograph of the whole experimental setup is shown in Fig. 3 (b).

![Configured experimental setup](image1)

![Photograph](image2)

Fig. 3. Experimental setup. (a) Configured experimental setup (b) Photograph.

The PA is produced by Tokin Company of Japan and has the dimensions of $5\times5\times20$ mm. A capacitor-type gap sensor (Micro Sense: 3401-RA2) owns the measuring range $\pm25/250$ $\mu$m and the corresponding precision is $10/100$ mm. The input and output displacements of the SR amplifying mechanism can be measured by the capacitor-type gap sensor. The waveform of the applied input voltage is generated by the LABVIEW, which is employed to communicate with the digital and analog signals during experimental process. The DA/AD converter (PCI-6052E) with a resolution of 12 bits is used to transform the waveform to the power amplifier (Chichibu Onoda), which has the voltage output range $\pm200$ Volts.

III.1.3 Parameter Identifications

The physical model of the SR amplifying mechanism driven by the PA is described by Eqs. (3) and (4). In the parameter identification, the structure material can be assumed as elasticity and set $n=1$ in Eq. (3) to combine with the dynamic equation (4) of the SR amplifying mechanism. We utilize the PSO method to identify the 7 parameters $m$, $c$, $k$, $d_e$, $\alpha$, $\beta$, and $\gamma$ simultaneously, and the fitness function can be described as Eq. (5a). Alternatively, we also could set $n \neq 1$ in Eq. (3) and identify the 7 parameters simultaneously.

III.1.4 Comparisons Between the PSO and RGA Results

Figure 4 shows the displacement errors with the PSO and RGA methods of the SR amplifying mechanism for Bouc-Wen hysteresis models. The identified value for the
Bouc-Wen hysteresis models are shown in Table I.

Fig. 4. The displacement errors with the PSO and RGA methods of the SR amplifying mechanism for Bouc-Wen hysteresis models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>m (kg)</th>
<th>c (Ns/m)</th>
<th>k (N/m)</th>
<th>d_e (m/V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGA</td>
<td>0.1</td>
<td>60</td>
<td>4486</td>
<td>2.171e-7</td>
</tr>
<tr>
<td>PSO</td>
<td>0.1</td>
<td>60</td>
<td>4497</td>
<td>2.174e-7</td>
</tr>
<tr>
<td>Parameter</td>
<td>α</td>
<td>β</td>
<td>γ</td>
<td></td>
</tr>
<tr>
<td>RGA</td>
<td>0.195</td>
<td>0.005</td>
<td>0.0222</td>
<td></td>
</tr>
<tr>
<td>PSO</td>
<td>0.2112</td>
<td>0.005</td>
<td>0.0299</td>
<td></td>
</tr>
</tbody>
</table>

The identified values of the Bouc-Wen (n≠1) hysteresis model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>m (kg)</th>
<th>c (Ns/m)</th>
<th>k (N/m)</th>
<th>d_e (m/V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGA</td>
<td>0.1</td>
<td>53.089</td>
<td>4e3</td>
<td>2.17e-7</td>
</tr>
<tr>
<td>PSO</td>
<td>0.1</td>
<td>58.074</td>
<td>4499</td>
<td>2.168e-7</td>
</tr>
<tr>
<td>Parameter</td>
<td>α</td>
<td>β</td>
<td>γ</td>
<td></td>
</tr>
<tr>
<td>RGA</td>
<td>0.207</td>
<td>0.005</td>
<td>0.0249</td>
<td></td>
</tr>
<tr>
<td>PSO</td>
<td>0.198</td>
<td>0.005</td>
<td>0.0251</td>
<td></td>
</tr>
</tbody>
</table>

III.2 Control Gains in a Slider-Crank Mechanism System

III.2.1 Slider Crank Actuated by a PM Synchronous Motor

A model of a PM synchronous motor can be simplified and its block diagram is shown in Fig. 5.
Fig. 5. Block Diagram of a PM Synchronous Motor.

Usually, the PM synchronous motor is coupled with a gear speed reducer with a gear ratio of $n$. Hence, the applied torque can be described as [6]:

$$\tau_e = \tau_m + B_m \omega_r + J_m \dot{\omega}_r.$$  \hfill (8)

where $\tau_m$ is the load torque, $B_m$ is the damping coefficient, $\omega_r$ is the rotor speed and $J_m$ is the moment of inertia.

In this section, Hamilton's principle and Lagrange multiplier are used to derive the differential equation for the slider-crank mechanism, which is shown in Fig. 6 [6].

Fig. 6. A Slider Crank Mechanisms System.

This section establishes a dynamic model of the slider-crank mechanism using the Lagrange method [6]:

$$M(\Psi)\dddot{\Psi} + N(\Psi, \dot{\Psi}) - BU - D(\Psi) + \Phi^T \lambda = 0.$$ \hfill (9)

The translation position $X_b$ can be obtained by transforming $\theta$ as

$$X_b = r \cos \theta + l \cos \phi + l'$$

$$= r \cos \theta + \left(l^2 - r^2 \sin^2 \theta \right)^{\frac{1}{2}} + l'$$ \hfill (10)

where $r$ and $l$ are the lengths of the crank and rod; $\theta$ and $\phi$ are angles of the crank and rod and $l'$ is half length of slider.

III.2.2 Experimental Results

In order to demonstrate the proposed control rules, a PC-based experimental equipment is setup in this section. The experimental instrument of the slider-crank mechanism is divided into three parts: the actuator; the slider crank; and the controller. The photographic representation is shown in Fig. 7.
The first part consists of a PM Synchronous motor and a driver. The driver is worked on 3-phase, 220 V, and 60 Hz. The slider-crank mechanism is coupled with the PM motor. The translation position is measured by a photometer. The output of the photometer scalar is 20000 pulses/m, and when mapped to real translation position, is 0~0.11 m. In order to carry out the parameter varying and external load an external mass (0.8 kg) is added onto the slider. The controller is based on a PC with Pentium-586 CPU. The PC plays the role of software development and data process. The data acquisition interface card (Advantech CO., PCL-1800) is installed in the ISA bus so as to handle the A/D and D/A processes. The graphical software of Simulink is used to implement the proposed control rule. At the same time, the linear converts between the physical scale and voltage from sensors are also determined by this software.

Based on the same requirements of simulations, the experimental results were acquired from the HP 54601B 100 MHz 4 channel oscilloscope. Firstly, the fixed PID setup under no-load is experimented. Under no-load conditions, the experimental results are shown in Fig. 8(a). However, the responses shown in Fig. 8(b) have a large overshoot when the load is added. It shows the bad robustness of the fixed PID controllers. The second experiment is under the fixed PID controllers whose parameters are obtained under full-load conditions. Figure 8(c) shows the experimental results. It shows that manually-chosen parameters can achieve desired requirements under full-load conditions. However, Fig. 8(d) shows the same controller applied when the load is removed. Obviously, the fixed PID controller cannot overcome the large change in operating situations.
Finally, by applying the proposed self-tuning PID controller to the same experimental instrument, the responses of translation position with no-load conditions and are shown in Fig. 9(a). It can achieve the same performance with the fixed PID controller under no-load conditions. The responses with full-load are shown in Fig. 9(b). The dynamic responses are almost the same with those of no-load conditions. It also proves that the proposed self-tuning PID controller has great robustness in the presence of parameter variations and external loads.
Fig. 9. Response of translation position with self-tuning PID control

(a) Experimental result with no-load.  (b) Experimental result with 0.8 kg load

III.3 The Optimal-Damped of a 3RPS Parallel Manipulator

III.3.1 Dynamic of a 3RPS Parallel Manipulator

Figure 10 shows the 3RPS parallel manipulator, one end is a moving platform, and the other end is a fixed base.

Fig. 10. Coordinate systems for the 3RPS manipulator.

The moving platform is connected to the fixed base with three limbs via three spherical joints \( (B_1, B_2, B_3) \) and three revolute joints \( (A_1, A_2, A_3) \). Each limb consists of two links connected in series with prismatic joint. It is seen in Fig. 10 that the origin \( P \) of the coordinate system \( [x \ y \ z]^T \) are located at the mass center of the base moving platform. The Cartesian coordinate vector \( [X \ Y \ Z \ \alpha \ \beta \ \gamma]^T \) denotes the fixed coordinates, where \( [X \ Y \ Z]^T \) and \( [\alpha \ \beta \ \gamma]^T \) denote, respectively, the vectors of positions and orientations. The fixed and moving platforms of the 3RPS are circles with radius \( a \) and \( b \), respectively. The distance between the three spherical joints on the moving platform is \( w \).
Let us focus on the mass center $O_i$ of the $i$th link as shown in Fig. 10. There are three components of the weight force of the moving platform acting on the $i$th link as $F_{pix}$, $F_{pix}$, $F_{pix}$, the weight force of the $i$th link is $W_i$, the acting force of the separately-excited DC servo motor on the revolution parts is $F_{1\text{motor}}$, and the acting force on each vertical leg is $F_{2\text{motor}}$. The 3RPS parallel manipulator dynamic motion of the links and moving platform is described by using Newton-Euler equations, which are formulated by considering the free-body diagrams of the links and moving platform separately [35-36]. Through reduction and incorporation, we can calculate the acting force on each vertical leg by using the following force equation for the moving vertical leg [36]:

$$F_{2\text{motor}} = F_{pix} + R_i W_{lix}.$$

where

$$R_i = \begin{bmatrix}
\cos \phi \cos \psi \cos \theta - \sin \phi \sin \psi - 
\cos \phi \cos \theta \sin \psi - \sin \phi \cos \psi & \cos \phi \sin \theta \\
\sin \phi \cos \psi \cos \theta + \cos \phi \sin \psi - 
\sin \phi \cos \psi \sin \phi + \cos \psi \cos \phi \sin \phi \sin \theta & \cos \phi \sin \theta \\
\sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta
\end{bmatrix}.$$

About the revolution parts, we have the equation for the first motor:

$$l_{a1} i_{a1} + r_{a1} i_{a1} = V_T - k_{b1} \omega, \quad \tau = K_{b1} i_{a1}.$$

where $K_{b1}$ is the torque coefficient, $k_{b1}$ is the back emf of the motor, $l_{a1}$ is the armature inductance, $r_{a1}$ is the armature resistance, $V_T$ is the terminal voltage, $\omega$ is the rotor speed and $i_{a1}$ is the armature current.

**III.3.2 Experimental Results**

The physical model is shown in Fig. 11.
The moving platform of the manipulator is a circle plate with radius $b = 22.08$ cm and the distance between any two of three spherical joints on the moving platform is $w = 36.16$ cm. The fixed-base is an inscribed triangle within a circle with radius $a = 41.72$ cm. The lengths of the three limbs are represented by $d_i (i = 1, 2, 3)$, and $\theta_1, \theta_2$ and $\theta_3$ are the rotating angles of the revolute joint on the first, second and third limbs, respectively. A numerical program is written to calculate the singular point, it is found that when $d_1 = 0.0717$ cm, $d_2 = 9.7488$ cm and $d_3 = 0.0771$ cm, and $\theta_1 = 19.9834^\circ$, $\theta_2 = 0.2438^\circ$ and $\theta_3 = 19.9074^\circ$, one singular point exists in the workspace, that is, the center coordinate of the moving platform with respect to the fixed base is $(1.6144, -2.7933, 0.0308)$, and the unit is cm. In this section, the point $P$ is required to move at a constant speed from the start point $(0, 0, 0)$, passes through the singular point $(1.6144, -2.7933, 0.0308)$, and arrives at its final point $(5, -5, 5)$.

From our trials, the PSO method in searching the damping factors. Seven optimal pairs of acceleration-damping factors with $(19.7621, 0.0151), (25, 0.02), (22.8946, 0.01213), (19.6683, 0.00649), (16.633, 0.0027), (14.2834, 0.00088), (12.5, 0.00022)$ are used to construct the GM(1,2) model, where the relative development coefficient, the relative influence coefficient, and the grey control coefficient are $b_1 = 0.0773$, $b_2 = -0.4071$, and $U = 32.8857$, respectively.

Figures 12 (a-f) show the experimental results of the trajectory and velocity in the X-axis, Y-axis and Z-axis with the HDRAC with a fixed damping value and the HDRAC with the PSO and GP controllers. It is found that the latter can achieve very good performance, and the system approximates stable and has very small fluctuations. These experimental results demonstrate that the HDRAC overcomes the fluctuation problem when the manipulator arrives at the singular point. It also proves that the proposed HDRAC with the PSO and GP controllers has great robustness.
VI. Conclusion

This paper presents in detail the background and implementation of a particle
swarm optimization algorithm suitable for constraint structural optimization tasks. The effectiveness of the approach is illustrated by three examples. Results show the ability of the proposed methodology to find better optimal solutions for structural optimization tasks than other optimization algorithms and provide a better understanding of the dynamic of the PSO, and provide a valuable guideline for selecting control parameters.

References


