Design of Optimal Coasting Speed for Saving Social Cost in Mass Rapid Transit Systems

Hui-Jen Chuang, Chao-Shun Chen, Chia-Hung Lin, Ching-Ho Hsieh and Chin-Yin Ho

Abstract—The artificial neural network (ANN) has been proposed in this paper to determine the optimal coasting speed of the train set for a mass rapid transit system to achieve the maximization of social welfare. The energy consumption and the traveling time to complete the journey between stations with various riderships are calculated by executing the train performance simulation to generate the data set for ANN training. The objective function is formulated by considering the cost of energy consumption and the cost of passenger traveling time. The ANN model is obtained after performing the ANN training, which can be applied to solve the optimal coasting speed of train sets according to the distance between stations and the ridership of passengers. To demonstrate the effectiveness of the proposed ANN model, the forecasted annual ridership of train sets for Kaohsiung Mass Rapid Transit (KMRT) system is used to determine the optimal coasting speed of train sets operating between stations for each study year. The corresponding profile to determine the optimal coasting speed of train sets operating sets for Kaohsiung Mass Rapid Transit System (KMRT) has been constructed since 1999 and it will be commissioning in 2007. To achieve the maximization of social welfare, the optimal coasting speed of train sets between stations should be determined by considering the cost of passenger traveling time and the cost of propulsion energy of the train set.

I. INTRODUCTION

To provide the public transportation function with high passenger carrying capacity, mass rapid transit systems (MRT) have been implemented in many metropolitan regions to solve the problems of traffic congestion and air pollution effectively. The MRT systems have become an important infrastructure for more and more large cities in the whole world. With the increase of electricity energy cost, the energy conservation of train set operation becomes a very critical issue for the mass rapid transit systems to pursue higher operation efficiency. With the operation of Taipei Mass Rapid Transit System for more than 10 years, people enjoy the convenient and comfortable traveling provided by the MRT system. On the other hand, Kaohsiung Mass Rapid Transit System (KMRT) has been constructed since 1999 and it will be commissioning in 2007. To achieve the optimization of overall social welfare and the effectiveness of energy saving for the operation of KMRT systems, it is necessary to design the proper coasting speed planning in advance. According to the operation of a typical MRT system, the energy consumption may contribute more than 20% of total operation cost and the propulsion power consumption of train sets is about 75% of the total power consumption of the MRT system. By operating the train sets with coasting control, the energy consumption of traction drive system can be reduced by 30%, while the traveling time to complete the journey between stations will be increased too [1]. It is the general practice to obtain the energy conservation by proper coast speed planning providing that the increase of passenger traveling time is acceptable. To achieve the maximization of social welfare, the optimal coasting speed of train sets between stations should be determined by considering the cost of passenger traveling time and the cost of propulsion energy of the train set.

To solve the optimal coasting planning for train operation, the cost of energy consumption and the cost of traveling time for passengers to complete the journey at each possible coasting speed have to be derived and included in the data set for the training of ANN network. By applying the ANN model developed, the coasting speed and the speed profile of a train set operated between stations can therefore be determined according to the track alignment and passenger ridership. Up to now, ANN has been successfully used to solve the optimization problem by imitating the human neural network. It has been applied in the areas of function approximation and pattern recognition [2]. It is especially suitable for the multivariable applications because of the capability to identify the interaction between the inputs and outputs. Based on the forecasting of passenger ridership, the coasting speed planning for KMRT system has been determined for each study year to obtain the maximization of social welfare. The train performance simulation (TPS) [3-5] has been applied to solve the energy consumption of train set by considering the constraint of speed restriction, the distance between stations, passenger loading factor, traction effort, operation mode and various types of operation resistance. The cost of energy consumption and the cost saving of traveling time of passengers are calculated by executing the train performance simulation for different scenarios of riderships, distance between stations and the time value of passengers [6,7] to generate the data set for ANN training. With the ANN model derived, the optimal coasting speed of the train set traveling between stations is determined according to the input neurons of ridership and track alignment. To demonstrate the effectiveness of the proposed ANN model, the distance

H. J. Chuang and C. Y. Ho are with the Department of Electrical Engineering, Kao Yuan University, Lu Chu, Taiwan, China (e-mail: hjchuang@cc.kyit.edu.tw)

C. S. Chen and C. H. Hsieh are with the Department of Electrical Engineering, Sun Yat-Sen University, Kaohsiung, Taiwan, China (e-mail: escchen@mail.ee.nsysu.edu.tw)

C. H. Lin is with the Department of Electrical Engineering, Kaohsiung University of Applied Sciences, Kaohsiung 807, Taiwan, China (chlin@mail.ee.kuas.edu.tw)
between stations, speed restriction, annual ridership and time value of passengers for the Orange Line of KMRT system is considered as the input neurons. The optimal coasting speed with the corresponding power consumption profile has been solved by applying the ANN network to minimize the social cost of MRT system operation.

II. PROBLEM DESCRIPTION

A. Description of KMRT System

The KMRT network consists of the Orange Line and Red Line with total track lengths of 14.4 km and 28.3 km respectively. The Red Line route serves the corridor from south to north while the Orange Line serves the corridor from west to east of Kaohsiung metropolitan area. There are 38 train stations, one main depot, and two subdepots. Kaohsiung City Government awarded a Build Operate and Transfer (BOT) contract to Kaohsiung Rapid Transit Corporation with concession period of 35 years. The construction has been started since 2001 with the passenger service to be commissioned by the end of 2007.

B. Mathematical model of speed profile

The propulsion traction effort and various types of operation resistance of the train set traveling between stations are calculated according to the characteristics of rolling stocks, passenger loading factor, operation mode and track alignments of the main line such as route gradient and curvature. The motion equation of train sets has been applied to find the mechanical power demand and the speed of each train set along the main line for each time snapshot of train performance simulation.

1) Train motion equation

To solve the power consumption of train operation at each snapshot, it is necessary to derive the motion equation of the train set along the main line so that the traction effort required can be simulated. The applied traction effort \( F_a \) of the train set will be used to generate the acceleration effort \( F_a \) for the train set and to overcome the operation resistance \( F_s \) [3].

\[
F = F_a - F_s = Ma
\]  
\( (1) \)

The acceleration of a train set is determined by the operation modes in each region with the speed solved by Euler’s method.

\[
V_{n+1} = V_n + 3.6a\Delta T \quad \text{(km/h)}
\]  
\( (2) \)

where \( \Delta T \) is the simulation time interval and \( V_n \) is the corresponding train speed for time snapshot \( n \).

After determining the train speed, the trapezoidal rule is applied to find the new position \( S_{n+1} \), of the train set according to the previous position \( S_n \).

\[
S_{n+1} = S_n + \frac{1}{2} (V_n + V_{n+1}) \frac{\Delta T}{3.6} \quad \text{(m)}
\]  
\( (3) \)

For each time snapshot, the applied traction effort is calculated by (1) according to the acceleration, speed and total weight of the train set between two stations. The mechanical power output of the induction motors is therefore solved.

\[
P_{\text{me}} = F_a \cdot V
\]  
\( (4) \)

The electric power output of the VVVF inverter to drive the train set is determined by considering the efficiency of the induction motors and gear boxes.

\[
P_{\text{me}} = P_{\text{out}} / (\eta_{\text{motor}} \cdot \eta_{\text{gear}})
\]  
\( (5) \)

where \( \eta_{\text{motor}} \) and \( \eta_{\text{gear}} \) are the operation efficiency of the induction motors and gear boxes.

C. Speed profile simulator

To calculate the performance of train movement on the main line of an MRT system, the train performance simulation system was developed in this paper. It mainly consists of train movement simulator and power consumption simulator. The weight and ridership of the train set, the alignment of main lines such as locations of train stations, grade, curvature, and speed restriction are collected as the input data for the computer simulation. The motion equations of the train set describing the speed and time versus distance along the main line is derived to solve the maximum performance run. The train movement simulator can generate the running speed, acceleration, location and the traction effort of the train set for each time snapshot. The power consumption and regeneration of the train set can be calculated by executing the power consumption simulation according to the traction effort and speed at each time snapshot. The core of proposed program for the maximum performance run is embedded in the forward and backward trajectory calculations. In the forward calculations, the equation of motion is integrated in the direction of positive time or forward distance flow, while in the backward calculations, it is integrated in the direction of negative time or backward distance flow. With speed restriction at same specifies locations between stations, the ANN training data set has been generated by considering the speed constraint in the train performance simulation.

For the operation of train set between stations O13-O14 of Orange Line with distance of 1035 m, the speed restriction of 64 km/h from 337 m to 525 m is applied and the maximum operating speed of 80 km/h is used for the other sections without speed restriction. Fig. 1 shows the traction effort and speed profile of a train set operating between stations O13 and O14. The range of constant traction effort or constant torque is represented by the straight line from knee point 0 to 1, which corresponding to the operation of train set starting from the station to the speed up to 40 km/h. The constant power operation mode is represented by the range of knee point 1 to knee point 2-1 in which the traction effort is proportional to the inverse of train speed from 40 km/h to 60 km/h. After the train speed reaches 60km/h, the train set is operated with the constant slip frequency and the traction effort is inversely proportional to the square the train speed. To comply to the speed constraint, the train speed is reduced from 80km/h to 64km/h with electric regeneration braking from 236m to 337m. The train set then is operated with cruising mode from 337m to 525m and only small input propulsion power is required. From 525m to 603m, the train speed is increased from 64km/h to 80km/h and the coasting mode is applied from 603m to 796m by turning off the propulsion power. When the train approaches the next station, the electric regeneration braking
is used by operating the induction motors as induction generators to achieve the energy conservation by converting the kinetic energy of the train set into electricity energy for restoration.

Table II shows the traveling time and energy consumption of a train set operated between stations O13 and O14. The traveling time to complete the journey is 72.1 seconds. The energy input for propulsion, energy regeneration by electric braking and the net energy consumption by the train set are 18.64 (kWh), 9.98 (kWh) and 8.49 (kWh) respectively.

### Table II

<table>
<thead>
<tr>
<th>Traveling time (sec)</th>
<th>Weight (ton)</th>
<th>Energy for propulsion (kWh)</th>
<th>Energy regeneration (kWh)</th>
<th>Net energy consumption (kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>72.1</td>
<td>177.6</td>
<td>18.64</td>
<td>9.98</td>
<td>8.49</td>
</tr>
</tbody>
</table>

### D. Objective function

To design the optimal coasting speed of the train set operated between stations of an MRT system, the objective function is formulated by including the cost of energy consumption $C_{SE}$ and the equivalent cost of passenger traveling time $P_{SC}$ to complete the journey in (6). The energy consumption of each train set with different ridership and coasting speed can be derived by executing the train performance simulation. After performing the neural network training, the ANN model is therefore obtained, which can be used to solve the optimal coasting speed of train sets.

\[
F_{SE} = C_{SE} + P_{SC} 
\]

1) Equivalent cost of passenger traveling time

The equivalent cost of passengers traveling time is calculated by multiplying the traveling time of all passengers to complete the journey between stations and the unit time cost [7]. By comparing to the traveling time required for the coasting speed of 80 km/h, the traveling time cost of passengers at different coasting speed will be increased in (7).

\[
P_{tc} = \frac{t_c \cdot W \cdot \Delta t}{3600} 
\]

, where $t_c$ is the unit time cost of passengers ($/h$). $W$ and $\Delta t$ are the total passengers on the train set and the increase of traveling time (sec) respectively.

2) Propulsion power consumption of MRT train sets

The energy consumption cost of a train set operated between stations with different coasting speed is calculated according to the reduction of energy consumption multiplied by the unit energy cost as expressed in (8).

\[
C_{SE} = \Delta E \cdot C_E 
\]

, where $\Delta E$ and $C_E$ are the reduction of energy consumption (kWh) and unit energy cost ($/kWh$) respectively.

### E. Training data set

To generate the training data set for the ANN model to solve the optimal coasting speed of train set with speed restriction, the operation of a train set between stations O13-O14 of Orange Line of KMRT has been selected for computer simulation. Fig. 3 shows the traveling time cost of passengers, the cost of energy consumption with coasting speed of the train set from 65 km/h to 80 km/h. The power consumption of the train set is solved as the product of the traction effort and speed. With the increase of coasting speed, the corresponding
energy consumption of train set will be increased too. By applying the lower coasting speed, the traveling time to complete the journey will be longer to result in the larger traveling time cost of passengers. For the same coasting speed, the traveling time cost is increased with the unit time value of passengers. On the other hand, the cost of energy consumption of the train set will be reduced linearly with the decrease of coasting speed.

Figure 4 shows the profiles of the social cost which consists of the cost of energy consumption and the cost of passenger traveling time. The social cost is calculated by equation (6) according to different unit time values of passengers and coasting speeds. At the same coasting speed, the reduction of social cost is decreased with the increase of the time value of passengers. Besides, at the same time value of passengers, the reduction of social cost is decreased with the increase of coasting speed. For instance, the optimal coasting speed is solved as 67 km/h when the time value of passengers is NT$60/hour and the maximum value of social cost for each passenger is NT$5.69/hour.

To solve the ANN model for the design of optimal coasting speed of an MRT system, the learning data set has been generated first by solving the objective function with different traveling time value of passengers and different riderships in Fig.5. It is found that the optimal coasting speeds will be varied with different time values of passengers for the same ridership. By increasing the ridership of the train set or the time value of passengers, the optimal coasting speed will be increased to achieve the best social welfare. For example, when the time value of passengers is NT$60/hour and the ridership is 423 passengers, the optimal coasting speed of 80 km/h should be applied for the train operation. Besides, when the time value of passengers is NT$40/hour, same optimal coasting speed of 73 km/h should be considered for the ridership between 335 passengers and 364 passengers with different social cost reductions resulted.

F. Headway of train operation

With the growth of ridership over the study years, the headways of train operation have to be adjusted to meet the criterion of service quality. The loading factor (LF) of train sets and operation headway of each transit route during the peak period is derived according to the ridership forecast over the study years. Fig. 6 shows the headway and passenger loading of each train set between stations R10-R11 of the Red Line and O13-O14 of the Orange Line over the study years. For the Red Line, the headways of train operation have to be reduced at years 2008, 2009, 2010 and 2013 with the minimum headway of 2 minutes after year 2013. By the same way, the headway of the Orange Line is reduced with the increase of ridership.

III. TRAINING ALGORITHMS OF THE BACK-PROPAGATION ARTIFICIAL NEURAL NETWORKS

A multi-layer feed-forward ANN network with back-propagation algorithm has been adopted for the ANN training in this paper. The data is propagated from the input layer to the hidden layers before reaching the final output layer. The error signals at the output layer are then propagated back to
both the hidden layers and input layer. The sum of square error (SSE) is then minimized by adjusting the synaptic between two layers and bias in any layer during the training process of ANN models as shown in Fig. 7. For a multi-layer network, the net input \( y^{k+l}(i) \) and output \( y^{k+l}(i) \) of neuron \( i \) in the \( k+l \) layer can be expressed as (9) and (11) respectively.

\[
\Delta = E^T E(\tau) \quad \text{(14)}
\]

\[
\alpha = \sum_{j} \psi_j(\tau) \quad \text{(15)}
\]

\[
E(\tau) = E(\tau) \quad \text{(16)}
\]

\[
\psi = \frac{1}{N} \sum_{j} \psi_j(\tau) \quad \text{(17)}
\]

\[
\psi_2 E(\tau) = J^T (\tau) \quad \text{(18)}
\]

\[
J(\tau)_{\tau} = \left[ \begin{array}{c}
\partial \mathcal{E}(\tau) / \partial \theta_{11} \\
\partial \mathcal{E}(\tau) / \partial \theta_{21} \\
\vdots \\
\partial \mathcal{E}(\tau) / \partial \theta_{M1} \\
\end{array} \right] \\
\left[ \begin{array}{c}
\partial \mathcal{E}(\tau) / \partial \theta_{12} \\
\partial \mathcal{E}(\tau) / \partial \theta_{22} \\
\vdots \\
\partial \mathcal{E}(\tau) / \partial \theta_{M2} \\
\end{array} \right] \\
\vdots \\
\left[ \begin{array}{c}
\partial \mathcal{E}(\tau) / \partial \theta_{1N} \\
\partial \mathcal{E}(\tau) / \partial \theta_{2N} \\
\vdots \\
\partial \mathcal{E}(\tau) / \partial \theta_{Mn} \\
\end{array} \right]
\]

(19)

\[
S(\tau) = \sum_{j} \psi_j(\tau) E(\tau) \quad \text{(20)}
\]

\[
N = R \times SK \quad \text{. The synaptic weights and biases are adjusted by (21)}
\]

\[
\Delta \tau = \left[ J^T (\tau) J(\tau) + \mu I \right]^{\dagger} J^T (\tau) R(\tau) \quad \text{(22)}
\]

where the parameter \( \mu \) is updated according to the change of the performance index for each iteration of the training process and \( I \) is the identity matrix. For the LMBP algorithm, all the input/output vector pairs \( R \) are given to the network first, and the corresponding outputs and errors are computed by using (11), (12) and (14). The Jacobian matrix \( J(\tau) \) in (19) and the incremental change \( \Delta \tau \) in (22) are calculated and the performance index is updated with new operating point of \( \tau + \Delta \tau \). The ANN becomes converged if the performance index is less than the specified tolerance. Otherwise, the parameter \( \mu \) is modified to repeat the training process of the neural network.

IV. OPTIMAL EXPANSION PLANNING OF TRACTION SUBSTATION CAPACITY

This section presents growth of the process to determine the optimal coasting speed for case studies of KMRT system with the annual ridership by using the ANN with LMBP training algorithm. The ANN model has been derived as described in the previous sections for the design of annual optimal coasting speed for the KMRT system. According to the distance between stations, the annual ridership forecast and the unit traveling time value of passengers, the optimal coasting speed of train set between stations of an MRT system can be derived in a very effective manner. With the track alignment information of different sections of Orange Line of KMRT system, various speed limits has been applied on some specific locations along the main lines to ensure the safety of train operation. To demonstrate the effectiveness of the proposed methodology to determine the optimal coasting speed of train sets for running at a speed restriction, Section between stations O13 and O14 of Orange Line has been selected for computer simulation. The distance between these two stations is 1,035 m with the speed restriction of 64 km/h between 337 m and 525 m from station O13. The annual ridership of different sections and operation headway of each transit route during the peak and off peak periods are derived according to the ridership forecast of KMRT system.
In this study case, the time values of passengers and the annual ridership between stations are selected as the input values of ANN model for the computer simulation. For the operation of train sets at peak period, the optimal coasting speed of KMRT system has been solved as shown in Fig. 8. The headway is reduced with the growth of ridership as described previously. It is found that the annual optimal coasting speed is varied with the change of passenger ridership of the train set. The optimal coasting speed is increased with the increase of unit time value of passengers.

![Fig. 8. The optimal coasting speed and passengers loading of train set for the peak period.](image)

Figure 9 shows the convergence of optimal coasting speed of train set for the unit passenger time value of NT$40/hour. The optimal coasting speed of train set has been obtained after 20 iterations of ANN simulation with error characteristics of 0.00047.

![Fig. 9. Convergence of optimal coasting speed of train set for the unit passenger time value of NT$40/hour.](image)

### Table III

<table>
<thead>
<tr>
<th>Traveling Time Value of Passengers (NT$h$)</th>
<th>Reduction of Energy Consumption Cost (NT$M$)</th>
<th>Increase of Passenger Traveling Time Cost (NT$M$)</th>
<th>Social Cost Reduction (NT$M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NT$40/h</td>
<td>100</td>
<td>51</td>
<td>49</td>
</tr>
<tr>
<td>NT$50/h</td>
<td>59</td>
<td>25</td>
<td>34</td>
</tr>
<tr>
<td>NT$60/h</td>
<td>39</td>
<td>15</td>
<td>24</td>
</tr>
</tbody>
</table>

Table III shows the reduction of energy consumption cost, the traveling time cost of passengers and the resultant social cost reduction over study periods of 30 years by applying the coasting speed derived for different traveling time values of passengers. With the increase of unit traveling time value of passengers, larger coasting speed will be used to save the passenger traveling time, which will introduce higher energy consumption cost.

### V. Conclusion

In this paper, the artificial neural network (ANN) has been used to determine the optimal coasting speed of train sets for an MRT system to achieve the reduction of social cost. The objective function is formulated by considering the cost of energy consumption of train sets and the traveling time cost of passengers. The energy consumption and the traveling time to complete the journey between stations with different riderships have been calculated by train performance simulation. According to the variations of traveling time values of passengers, energy charge rate, the ridership of train sets and the distance between stations, the optimal coasting speed of the minimum social cost is derived to generate the training data set of ANN model.

To demonstrate the effectiveness of the proposed ANN model to solve the optimal coasting speed of train set on the main line, the Orange Line of KMRT system has been selected for computer simulation. By applying the ANN model derived, the optimal coasting speeds of train sets operated between two stations have been determined according to the alignment of main lines and the annual ridership forecast over the study years. Although the traveling time cost of passengers to complete the journey is increased with the implementation of optimal coasting speed, the reduction of energy consumption cost will be more significant and economically justified. With the dynamic variation of passenger loading factors for the daily MRT operation, the ANN model derived can be used to determine the optimal coasting speed of train sets adaptively to enhance the operation efficiency of MRT systems by considering the costs of energy consumption and passenger traveling time. The ANN model derived does provide an important tool to design the proper operation strategy to achieve the minimization of social cost for the operation of MRT systems.

### VI. References

VII. BIOGRAPHIES

H. J. Chuang received the B.S. and M.S. degree in Electrical Engineering from Taiwan University of Science and Technology in 1990 and 1992 respectively, and Ph. D. degree in Electrical Engineering from Sun Yat-Sen University in 2002. He is presently an Associate Professor at Kao Yuan University, Lu Chu, Taiwan, China. His research interest is in the area of load flow and power system analysis of mass rapid system.

C.S. Chen received the B.S. degree from Taiwan University in 1976 and the M.S. Ph.D. degree in Electrical Engineering from the University of Texas at Arlington in 1981 and 1984 respectively. From 1984 to 1994 he was a professor of Electrical Engineering department at Sun Yat-Sen University. From 1989 to 1990, he was on sabbatical at Empros System International. Since Oct. 1994, he works as the deputy director general of Department of Kaohsiung Mass Rapid Transit. From Feb.1997 to July 1998, he was with the Taiwan University of Science and Technology as a professor. From August 1998, he is with the Sun Yat-Sen University as a full professor. His majors are computer control of power systems, electrical and mechanical system integration of mass rapid transit systems.

C.H. Lin received the B.S. degree from Taiwan Institute of Technology in 1991, M.S. degree from University of Pittsburgh in 1993, and Ph. D. degree in Electrical Engineering from University of Texas at Arlington in 1997. He is presently an Professor at Kaohsiung University of Applied Sciences. His area of interest is distribution automation and computer applications to power systems.

C. H. Hsieh received the B.S. degree in Kaohsiung University of Applied Sciences in 2004 and M.S. degree from Sun Yat-Sen University in 2006. His research interest is in the area of mass rapid system.

C.Y. Ho received the M.S. degree in mathematics from Cheng Kung University in 1993. She has been a lecturer in Kao Yuan University.