Stress singularity due to traction discontinuity on an anisotropic elastic half-plane

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Abstract

In a half-plane problem with $x_1$ paralleling with the straight boundary and $x_2$ pointing into the medium, the stress components on the boundary whose acting plane is perpendicular to $x_1$ direction may be denoted by $t_1 = [\sigma_{11}, \sigma_{12}, \sigma_{13}]^T$. Stress components $\sigma_{11}$ and $\sigma_{13}$ are of more interests since $\sigma_{12}$ is completely determined by the boundary conditions. For isotropic materials, it is known that under uniform normal loading $\sigma_{11}$ is constant in the loaded region and vanishes in the unloaded part. Under uniform shear loading, $\sigma_{11}$ will have a logarithmic singularity at the end points of shear loading. In this paper, the behavior of the stress components $\sigma_{11}$ and $\sigma_{13}$ induced by traction-discontinuity on general anisotropic elastic surfaces is studied. By analyzing the problem of uniform tractions applied on the half-plane boundary over a finite loaded region, exact expressions of the stress components $\sigma_{11}$ and $\sigma_{13}$ are obtained which reveal that these components consist of in general a constant term and a logarithmic term in the loaded region, while only a logarithmic term exists in unloaded region. Whether the constant term or the logarithmic term will appear or not completely depends on what values of the elements of matrices $\Omega$ and $\Gamma$ will take for a material under consideration. Elements for both matrices are expressed explicitly in terms of elastic stiffness. Results for monoclinic and orthotropic materials are all deduced. The isotropic material is a special case of the present results.

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1. Introduction

It is known that the stresses in a linear elastic body will in general possess singularities at points where the geometries of the boundary change abruptly, e.g., those tips of the cracks, or at junctions where different material properties met like those occurred in the free edges of bimaterial problems. There are other points of the...