Real-time interpolator and Contour Tracking in Link-space for a 3 DOF Parallel Mechanism

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Abstract. Parallel mechanisms could be hardly used in contour tracking because of their mechanism features. This study proposed a link-space real time contour tracking for a 3 DOF (Z - α and β) hydraulic parallel mechanism. The essence of this approach is to convert control points of command trajectory to link space by inverse kinematics. A real-time interpolator was created and the multi-axis cross-coupled pre-compensation control (MCCPM) was constructed for link-space contour tracking. It was shown that a contour-accurate trajectory tracking could be performed which was impossible in the original Z-α-β space. Other advantages of this link-space approach were time efficiency and the uniform tracking velocity.

1. Introduction

In order to improve the disadvantage of serial machine tools, the interest in the parallel mechanism intensified since 1980, in which parallel mechanism introduced by Stewart [1] was extensively discussed and further developed. The parallel mechanism is rigid, accurate and simple as compared with serial mechanism, but it is mathematically involved and susceptible to singularity problem. Perspectives and difficulties of parallel mechanisms were discussed in detail by Dasgupta and Mruthyunjaya [2]. A broad application of parallel mechanism is flight simulation. Six DOF flight simulators were the state of art but Nicolas and Clement [3] showed the performance of 3-DOF simulator and its cost advantage over 6-DOF flight simulators. A conceivable trend is to promote the parallel mechanism to the level of precision machine tool. However, traditional serial machine tools still have great margin in the ability of continuous trajectory tracking. The purpose of this study is to develop an approach, which enables a 3-UPU (Universal-Prismatic-Universal) hydraulic parallel mechanism with Z, α and β DOF as shown in figure 1 to do continuous trajectory tracking. The intention is that the parallel mechanism can be extended to a five axes machine tool by a combination with x-y table in the future.

Parallel mechanisms suffered from singular planes, which make the mechanisms incontrollable
or even get damaged. Gosselin and Angeles [4] analyzed inverse kinematics singularities (\(\text{det}(J_L) = 0\)), forward kinematics singularities (\(\text{det}(J_P) = 0\)) and combined singularities (\(\text{det}(J_L)=0\) and \(\text{det}(J_P)=0\)), Ma and Angeles’s analysis [5] was also based on Gosselin’s method. Zlatanov, Fenton and Benhabib [6] further distinguished between six types of singular configuration. In addition to the above disclosed singularities, Tsai [7] further illustrated that another forward kinematic singularity occurs when the geometry of the manipulator satisfies the following conditions: 1. The geometry of moving platform is identical to that of the fixed platform. 2. The manipulator assumes a configuration in which the limb lengths are equal to one another. 3. All the limbs are parallel to each other.

Since the upper and lower platform of the hydraulic parallel mechanism shown in figure 1 are of the same size, the type of singularities described by Tsai [7] lurk in its workspace. In order to exclude singularities from workspace, the mechanism is equipped with an auxiliary support (universal joint and pin joint). The upper platform can move in X, Z, \(\alpha\) and \(\beta\) direction but the movement in X is passive and constrained by the auxiliary support. Hence X is not an independent DOF. The DOF of the mechanism in figure is in Z, \(\alpha\) and \(\beta\).

Trajectory tracking by parallel platform, usually known as Stewart platform, is by no means straightforward. This is because although the positions of driving axes can be uniquely obtained by inverse kinematics, the position of centroid of the moving platform as calculated from given positions of driving axes is not unique. There can be as many as 16 solutions [8] for the centroid and Newton-Raphson’s algorithm [9] was often needed to get approximate values of those solutions. Impossible or incorrect values were obtained if the determinant of Jacobian matrix became zero during the computation [10]. In an effort to elude singular points, Perng and Hsiao [11] used damped least-squares method [12] to avoid the situation of \(\text{det}(J) = 0\), but the avoidance was accompanied by big position and orientation errors which ruined the performance of trajectory tracking.

This study proposed a link-space trajectory tracking in which the control points of trajectory were converted from Z-\(\alpha\)-\(\beta\) space to link-space by inverse kinematics and then a link-space trajectory was real-time generated by interpolation. An example shown in figure 2 may explain the link-space approach. In figure 2(a) a spatial trajectory in Z-\(\alpha\)-\(\beta\) space is to be tracked, the mathematical description of which is seen in section 4. Figure 2(b) is the trajectory projected on \(\alpha\)-\(\beta\) plane and figure 2(c) is the control points of trajectory converted to link-space. Figure 2(d) shows the link trajectories built from control points for each link. The individual link trajectories shown in figure 2(d) build “roots” for real-time interpolation and the tracking of the real-time interpolated trajectories creates the trajectory shown in figure 2(a) that cannot be created by the parallel mechanism in its own motion coordinates Z-\(\alpha\)-\(\beta\). The link-space approach not only makes the trajectory tracking possible, but also allows implementation of proven contour compensation strategies. Moreover, it is time efficient because the forward kinematics is exempted. This
enables real-time interpolation.

Poo et al. [13] pointed out that matched dynamics of x-y table would result low steady state and transient contour errors in tracking. Koren [14], Koren and Lo [15] proposed several planar cross-coupled compensation schemes. Tarng et al. [16] combined fuzzy feedrate controller with cross-coupled algorithm to control spatial trajectory on machine tools. ElBeheiry [17] proposed strategies have been further developed for either independent or cross-coupling backlash compensation. Chin and Tsai [18] proposed a pre-compensation system that can encompass other control schemes like adaptive control for robot-like serial machines. Chin and Lin [19] used the pre-compensation control for flexible robot arm. Chin and Lin [20] proposed a cross-coupled pre-compensation method (CCPM) for machine tools by integrating pre-compensation and cross-coupled compensation. Cheng and Chin [21] constructed a machine tool system model to investigate the ensemble effect of cutting, tracking (CCPM) and structure on machining errors. Fuzzy rules could be used to reduce computation while maintaining precision in tracking [22]. Lue et al.[23] was shown that a cross-coupled pre-compensation tracking control was possible for such a multi-axis machine tool. Cross-coupled tracking was shown also useful for other manufacturing process like rapid prototyping [24]. Recently, PC-Based computers became capable of industrial application in real-time interpolation jobs [25-26].

It would be valuable that such ability of spatial continuous trajectory tracking could also be developed for parallel machines on a real-time basis. A real-time interpolator based on [27] was constructed in this study and PC-based control was implemented.

2 System constructions

2.1 Kinematics and Mechanism

Figure 1 shows a parallel machine with an upper platform (U1, U2, U3, U4), a lower platform (B1, B2, B3, B4), connected by three hydraulic cylinders, seven universal joints, two pin joints and one supplementary support. There are three UPU mechanisms which are parallel individually. The movable platform P in figure 1 has three degrees of freedom: Z, α, and β with a constrained passive movement in X.

2.1.1 Inverse kinematics

Inverse kinematics is to transform the platform position and orientation to individual movement of hydraulic cylinders. The upper platform is in moving coordinates \((X_U, Y_U, Z_U)\), it is seen that

\[
^U U_i = \begin{bmatrix} R \cdot S \theta_i & R \cdot C \theta_i & 0 & 1 \end{bmatrix}^T; i = 1 \sim 3
\]

(1)

The U4 coordinates of upper platform in moving coordinates becomes

\[
^U U_4 = \begin{bmatrix} R \cdot C \theta_2 & 0 & 0 & 1 \end{bmatrix}^T
\]

(2)
where \( R \) is the radius of the outer circle, \( \theta_i = \pi + \frac{2\pi}{3}(i-1); i = 1 \sim 3 \), \( S, C \) means \( \sin() \), \( \cos() \) respectively.

The connecting points of the lower (fixed) platform in fixed coordinates \( (X_B, Y_B, Z_B) \) are

\[
^B B_i = \begin{bmatrix} R \cdot S \theta_i & R \cdot C \theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}; i = 1 \sim 3
\]  

(3)

and the \( B_4 \) coordinate lower platform in fixed coordinates are

\[
^B B_4 = \begin{bmatrix} R \cdot C \theta_1 & R \cdot S \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]  

(4)

The transformation matrix becomes

\[
^B T_U = \begin{bmatrix} C\gamma C\beta & C\gamma S\beta S\alpha - S\gamma C\alpha & C\gamma S\beta C\alpha + S\gamma S\alpha & X \\ S\gamma C\beta & S\gamma S\beta S\alpha + C\gamma C\alpha & S\gamma S\beta C\alpha - C\gamma S\alpha & 0 \\ -S\beta & C\beta S\alpha & C\beta C\alpha & Z \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]  

(5)

The above rotation matrix \( ^B T_U = T_{\gamma, \beta, \alpha} \) for roll, pitch and yaw can be specified in terms of rotations about the principal axes of the reference coordinate system and the rotating coordinate system [7].

The length of the supplementary support is

\[
|L_4| = \|^B B_4 - ^B B_4\| = \sqrt{\left(\|^B U_{4x} - ^B B_{4x}\|^2 + \left(\|^B U_{4y} - ^B B_{4y}\|^2 + \left(\|^B U_{4z} - ^B B_{4z}\|^2 \right)^2 \right)^2}
\]  

(6)

So \( X = (\|^B U_{4x} - ^B B_{4x}\| - \sqrt{L_4^2 - (\|^B U_{4y} - ^B B_{4y}\|^2 - (\|^B U_{4z} - ^B B_{4z}\|^2 \right)^2}) \)  

(7)

The connecting points of the upper platform are

\[
^B U_i = [^B T_U]^* [^U U_i]; i = 1 \sim 3
\]  

(8)

Therefore, the respective lengths of the driving links are

\[
|L_i| = \|^B U_i - ^B B_i\| = \sqrt{\left(\|^B U_{ix} - ^B B_{ix}\|^2 + \left(\|^B U_{iy} - ^B B_{iy}\|^2 + \left(\|^B U_{iz} - ^B B_{iz}\|^2 \right)^2 \right)^2}; i = 1 \sim 3
\]  

(9)

2.1.2 Forward kinematics

The link-trajectory required for Stewart platform can be uniquely obtained from inverse kinematics in form of link extensions, but the vice versa is not true. The determination of platform centroid position from link information is not unique.

The link extensions can be expressed as [11, 28]

\[
\delta L = J \delta \mathbf{P}
\]  

(10)
Jacobian can be calculated by calculating

$$J_{i1} = \frac{\partial L_i}{\partial X} = L_{ix}/L_i$$  \hspace{1cm} (11)$$

$$J_{i2} = \frac{\partial L_i}{\partial Z} = L_{iz}/L_i$$  \hspace{1cm} (12)$$

$$J_{i3} = \frac{\partial L_i}{\partial \alpha} = (L_{ix} RS\theta_i \cdot C\alpha \cdot S\beta + L_{iy} RS\theta_i \cdot -S\alpha + L_{iz} RS\theta_i \cdot C\alpha \cdot C\beta)/L_i$$  \hspace{1cm} (13)$$

$$J_{i4} = \frac{\partial L_i}{\partial \beta} = (L_{ix} R(C\theta_i \cdot -S\beta + S\theta_i \cdot S\alpha \cdot C\beta) + L_{iz} R(C\theta_i \cdot -C\beta + S\theta_i \cdot S\alpha \cdot -S\beta))/L_i$$  \hspace{1cm} (14)$$

where \( L_{ix} = U_{ix} - B_{ix} \); \( L_{iy} = U_{iy} - B_{iy} \); \( L_{iz} = U_{iz} - B_{iz} \); \( i=1\sim4 \);

The Jacobian is as follows:

$$J = \begin{bmatrix}
\frac{\partial L_1}{\partial X} & \frac{\partial L_1}{\partial Z} & \frac{\partial L_1}{\partial \alpha} & \frac{\partial L_1}{\partial \beta} \\
\frac{\partial L_2}{\partial X} & \frac{\partial L_2}{\partial Z} & \frac{\partial L_2}{\partial \alpha} & \frac{\partial L_2}{\partial \beta} \\
\frac{\partial L_3}{\partial X} & \frac{\partial L_3}{\partial Z} & \frac{\partial L_3}{\partial \alpha} & \frac{\partial L_3}{\partial \beta} \\
\frac{\partial L_4}{\partial X} & \frac{\partial L_4}{\partial Z} & \frac{\partial L_4}{\partial \alpha} & \frac{\partial L_4}{\partial \beta}
\end{bmatrix}$$  \hspace{1cm} (15)$$

From equation (10), the forward kinematics mapping can be expressed as

$$\partial \mathbf{P} = J^{-1}\partial \mathbf{L}$$  \hspace{1cm} (16)$$

Substituting link extensions \( \partial \mathbf{L} \) into equation (16), the position of movable plate P can be found, which is not unique [8]. Newton-Raphson algorithm [9] is usually needed to find the approximate solutions.

2.1.3 Singularity

Singularity is a severe problem for parallel mechanism. The mechanism may become statically indeterminate or even collapse along the singular plane. A look into the whereabouts of the singular points is indispensable before putting the parallel machine to work.

Gosselin and Angeles [4] used input-output relationship to analyze the singular points:

$$F(L, P) = \begin{bmatrix}
f_1(L, P) \\
f_2(L, P) \\
f_3(L, P) \\
f_4(L, P)
\end{bmatrix} = 0$$  \hspace{1cm} (17)$$

Differentiating equation (17) yields:
\[
\frac{dF}{dt} = \frac{\partial F}{\partial L} \frac{\partial L}{dt} + \frac{\partial F}{\partial P} \frac{\partial P}{dt} = -J_L \dot{L} + J_P \dot{P} = 0
\]  

(18)

\[J_P = \frac{\partial F}{\partial P} \quad \text{and} \quad J_L = -\frac{\partial F}{\partial L}
\]  

(19)

\[
\mathbf{k} = J_L^{-1} J_P \mathbf{k} = J\mathbf{k}
\]

(20)

hence equation (15) becomes \( J = J_L^{-1} J_P \).

Inverse kinematic singularity takes place if the determinant of \( J_L \) is zero [4], and forward kinematic singularity takes place if the determinant of \( J_P \) is zero. If both \( \det (J_L) \) and \( \det (J_P) \) are zero, it is combined singularity. Impossible or incorrect solutions may be obtained for \( \det (J) = 0 \) [10].

The mechanism in figure 1 is the type described by Tsai [7] but the auxiliary support excluded the problem of singularities.

2.1.4 Workspace

Workspace is the displacement space of maximum stretched links. Workspace of the parallel machine shown in figure 1 can be calculated by inverse kinematics of section 2.1.1. Figure 3 shows the workspace of the motion platform without auxiliary support while figure 4 shows the same but with auxiliary support. It is seen that workspaces of two rotated angles \((\alpha, \beta)\) do not change, but workspace in \(Z\) direction decreases from 400mm to 250mm. In return for the limitation of workspace, the mechanism with auxiliary support works without obstacles of singularity.

2.2 Real-time Interpolation and Multi-axis cross-coupled pre-compensation

A contour compensation and a uniform velocity control in the original \(Z-\alpha-\beta\) space is not executable because of the different units of original coordinates. However, once converted to the orthogonal coordinates of link-space as shown in figure 2(c), the control points can be on-line interpolated to become a link-space trajectory the motion platform tracked with uniform velocity and a contour compensation by multi-axis cross-coupled pre-compensation algorithm (MCCPM) becomes executable.

2.2.1 Real-time interpolator

The link-space trajectory was interpolated by 3\(^{rd}\) Hermite curve, which can be expressed as follows:

\[
P_i(u) = \begin{bmatrix}
    2 & -2 & 1 & 1
    -3 & 3 & -2 & -1
    0 & 0 & 1 & 0
    1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
P_i \\
P_{i+1}
\end{bmatrix}
\]

(21)

The tangent of Hermite curves, \( P'_i (i = 1, 2, \ldots, n - 1) \), can be calculated for free-end boundary
condition as follows:

\[
\begin{bmatrix}
2 & 1 & 0 & \Lambda & \Lambda & 0 \\
1 & 4 & 1 & 0 & \Lambda & 0 \\
0 & 1 & 4 & 1 & 0 & \Lambda \\
\Lambda & \Lambda & \Lambda & \Lambda & \Lambda & \Lambda \\
0 & \Lambda & \Lambda & 1 & 4 & 1 \\
0 & 0 & \Lambda & 0 & 1 & 2
\end{bmatrix}
\begin{bmatrix}
P_0' \\
P_1' \\
M \\
P_{n-1}' \\
P_n'
\end{bmatrix}
= 
\begin{bmatrix}
3P_1 - 3P_0 \\
3P_2 - 3P_0 \\
3P_3 - 3P_1 \\
M \\
3P_n - 3P_{n-1}
\end{bmatrix}
\]  
(22)

The parametric form of 3D (L1, L2, L3) curve can be represented as:

\[P(u) = L_1(u)w + L_2(u)w + L_3(u)w\]  
(23)

Once in parametric form, real-time interpolation becomes possible [27]. The feed rate along the curve \(P(u)\) is obtained by [27,29]

\[V(t) = \frac{dP(u)}{dt} = \frac{dP(u)}{du} \frac{du}{dt} \]  
(24)

Therefore

\[\frac{du}{dt} = \frac{V(t)}{\frac{dP(u)}{du} = \frac{V(t)}{\sqrt{(\frac{dP_1(u)}{du})^2 + (\frac{dP_2(u)}{du})^2 + (\frac{dP_3(u)}{du})^2}}} \]  
(25)

At \(t_k = kT\), the first-order Taylor’s expansion of equation (25) is

\[u_{k+1} \approx u_k + TV_k \]

\[u_{k+1} = u_k + \frac{TV_k}{\sqrt{(\frac{dP_1(u)}{du})^2 + (\frac{dP_2(u)}{du})^2 + (\frac{dP_3(u)}{du})^2}} \]  
(27)

where \(\frac{dP_1(u)}{du}, \frac{dP_2(u)}{du}, \frac{dP_3(u)}{du}\)

2.2.2 Contour errors

Spatial contour errors were computed according to [16]. Figure 5 explained the contour errors of an ideal trajectory in the link-space (L1, L2, L3). \(P_e\) is the position on trajectory that’s closest to the actual position \(P_a\). The position tracking error vector is defined as the distance between the actual and the desired centroid position of the upper platform:

\[E = [E_{L1}, E_{L2}, E_{L3}]^T = P_a - P_i \]  
(28)

On the other hand, the path contour error vector is defined as the shortest distance from the actual position to the desired trajectory:

\[E_{el} = [E_{el1}, E_{el2}, E_{el3}]^T = P_a - P_e \]  
(29)
The unit tangent velocity at desired position $P_i$ and actual position $P_a$ are $V_i = \frac{\mathbf{R}_i}{\|\mathbf{R}_i\|}$ and $V_a = \frac{\mathbf{R}_a}{\|\mathbf{R}_a\|}$ respectively. $\mathbf{V}$ is the unit linear velocity from $P_e$ to $P_i$. In closed loop control $E_r$ is approximately equal to $E_r'$ and the same is true for $V_i$ and $V_a$. Therefore $\mathbf{V}$ can be taken as the unit average velocity of these two velocities.

$$\mathbf{V} = \frac{\mathbf{R}_i + \mathbf{R}_a}{\|\mathbf{R}_i\| + \|\mathbf{R}_a\|} = \mathbf{V}_{L1} + \mathbf{V}_{L2} + \mathbf{V}_{L3}$$

(30)

The contour error can be obtained from $E_r = P_a - P_e = E - (P_i - P_e)$ as follows:

$$E_r \approx E_r' - (E \times \mathbf{V}) \times \mathbf{V} = E - (E \cdot \mathbf{V})\mathbf{V}$$

(31)

The contour error $E_r$ in terms of components in L1, L2, and L3 can be expressed as follows:

$$E_{rLi} = [E_{rL1}, E_{rL2}, E_{rL3}]^T = [E - (E \cdot \mathbf{V})\mathbf{V}] \cdot (\mathbf{v}_i, \mathbf{v}_j, \mathbf{v}_k)^T$$

(32)

$$\begin{cases}
E_{rL1} = \mathbf{V}_{L1}(E_{L1} \mathbf{V}_{L2} - E_{L2} \mathbf{V}_{L1}) + \mathbf{V}_{L3}(E_{L1} \mathbf{V}_{L3} - E_{L3} \mathbf{V}_{L1}) \\
E_{rL2} = \mathbf{V}_{L1}(E_{L2} \mathbf{V}_{L1} - E_{L1} \mathbf{V}_{L2}) + \mathbf{V}_{L3}(E_{L2} \mathbf{V}_{L3} - E_{L3} \mathbf{V}_{L2}) \\
E_{rL3} = \mathbf{V}_{L1}(E_{L3} \mathbf{V}_{L1} - E_{L1} \mathbf{V}_{L3}) + \mathbf{V}_{L2}(E_{L3} \mathbf{V}_{L2} - E_{L2} \mathbf{V}_{L3})
\end{cases}$$

(33)

$$|E_r| = [\sqrt{1 - \mathbf{V}_{L3}^2(E_{L1} \mathbf{V}_{L2} - E_{L2} \mathbf{V}_{L1})} + \sqrt{1 - \mathbf{V}_{L2}^2(E_{L1} \mathbf{V}_{L3} - E_{L3} \mathbf{V}_{L1})} + \sqrt{1 - \mathbf{V}_{L1}^2(E_{L2} \mathbf{V}_{L3} - E_{L3} \mathbf{V}_{L2})}]$$

(34)

2.2.3 Multi-axes Cross-couple pre-compensation method (MCCPM)

Cross-coupled pre-compensation method has been proven efficient for high speed, high curvature tracking [18-23], but past development was for serial machines. Figure 6 is the pre-compensation in link-space of the studied parallel machine. In figure 6, $\mathbf{v}_i$ is the unit tangent vector at machining point, $T$ is the tangent at $P_i$, $N$ is the unit normal vector at $P_i$, and $B = (T \times N)$ is the path direction. An on-line, real-time multi-axis cross-coupled pre-compensation system, shown in figure 7, was developed by integrating interpolation described in section 2.2.1 [27, 29] and contour error computation described in section 2.2.2.

In figure 7, the feeding or platform moving speed is given, which formed tangential velocity $V_t = V_b \mathbf{v}_i$, this will be used in the interpolator to generate reference points $L_{ai} (i = 1 \sim 3)$ for links. The feeding errors are:
\( e_{si} = L_{ij} - L_{ip}, \quad (i=1\sim3) \), where \( L_{ip}(i=1\sim3) \) are actual link length measured by potentiometer scales. The control signals to hydraulic servo valve, which make both position control and contour compensation, are

\[ U_{li} = K_{ei} \cdot e_{xi} + K_{ri} \cdot E_{ri}, \quad (i=1\sim3) \]

The actual lengths of hydraulic cylinders were measured by potentiometer scales and the contour errors were computed as described in section 2.2.2. Errors \( E_{ri} \) were used to perform cross-coupled compensation (CCS) and contour error \( E_r \) was used to perform pre-compensation, this made the structure become a cross-coupled pr-compensation control (CCPM). The essence of pre-compensation is a velocity modification of following form:

\[ V = V_i + V_k \]

where the feed velocity \( V_i \) is modified by a pre-compensation velocity \( V_k \), \( V_k = K_v E_r \), \( K_v \) is the velocity pre-compensation gain. The following is true for the proposed link-space maneuver:

\[ V = V_i + K_v E_r = [E_1, E_2, E_3] \]

3 Experimental Set-up

Figure 8 was the constructed hydraulic motion platform, the hydraulics of which contained three Parker D1FH proportional valves, three cylinders and power unit with filters, accumulator, and pressure regulators. Parker D1FH proportional valves are newly appeared hydraulic elements with a response frequency of 100 Hz. The valves receive control signals of ±10 V and have feedback features for spool position control. The radius of the outer circle is 300 mm. The initial lengths of the cylinders are 500 mm with extension capability of 400 mm, the outside diameters of pistons are 50 mm, outside diameters of rods are 35 mm, and each cylinder has a remarkable maximum force output of 1000 kgw. The cylinders are equipped with 10V potentiometer scale for extension length detection. Two CRS03 gyroscopes were installed to monitor rotational angles of the platform. The CRS03 gyroscope operates under 5VDC with rate range 0~±100degree/sec, the angle is obtained by multiplication of angular speed with time.

Figure 9 shows the empirical setup for the control of the hydraulic parallel platform, in which two control cards were used. Control card \( PCI\, 9111DG \) has 16 12-bit AD channels responsible for signals from three valves, two gyroscopes and three potentiometer scales, and one 12-bit DA channel. \( ACL\, 6128 \) has two 12-bit DA channel. Each DA channel produces ±10 V to control the proportion valve. The voltage range of potentiometer scale is 0V~10V (0~400mm) with the minimum resolution 1 bit=0.2mm \((2^{12} = \pm 10V)\). Experiments were controlled by program written in C++. Sampling time was 10 ms to cope with the 100 Hz valve frequencies.
4 Experiments and Discussions

4.1 Target Trajectory and Tracking Schemes

The trajectory to be tracked is described by
\[ Z = 20 \times tt \quad , \quad \alpha = r(tt) \times \sin(w(tt)) \quad , \]
\[ \beta = r(tt) \times \cos(w(tt)) \quad , \quad r(tt) = 0.05 \times tt \quad , \quad w(tt) = \pi / 30 \times tt \quad , \quad tt \text{ : time variable.} \]
Control points were taken by setting
\[ tt = 0 : 0.1 : 10 \quad \text{and shown in figure 2(a).} \]
After inverse kinematics the control points in link space were obtained as shown in figure 2(c), which gave three link trajectories as shown in figure 2(d) and three contour control schemes (US, CCS, CCPM) were implemented for comparison.

All three control schemes were accommodated by the structure in figure 7. Figure 7 described a multi-axis CCPM, if \( K_v \) was set zero, it became a CCS system, and if further \( K_r \) were also set zero, it was a US system. Note that US means no cross-coupling modification for contour errors, but it still performs position controls.

The purpose of experiments was to investigate the feasibility and performance of the real-time interpolated multi-axis cross-coupled precompensation system developed and shown in figure 7. For the sake of comparison gains were obtained according to [20 - 21] as \( K_{ex}=0.015 \), \( K_r=0.05 \) and \( K_v=0.01 \). Two feed rates were experimented: 50 mm/sec and 100 mm/sec. Comparison of contour errors was expressed in L1, L2 and L3 coordinates.

4.2 Results and Discussions

In the experiments the command trajectories shown in figure 2 (d) contained 100 control points, after real-time interpolation at rate 10 ms and tracking by respective link, the actual link trajectories were recorded in figures 10 – 15 (a) and the position errors were shown in figures 10 – 15 (b). Path contour errors as calculated using equation (32) were shown in figures 10 – 15 (c). The ultimate contour errors with respect to the target space trajectory were shown in figures 10 – 15 (d).

This shift in position error profile obviously brings out better contour precision. The path contour errors of respective link in figure 10(c) were improved as can be seen from the scales of figure 11(c) and 12(c). And contour errors of overall trajectory were improved from that shown in figure 10(d) to those in figures 11(d), 12(d).

At higher tracking speed, 100 mm/sec, the magnitude of position errors became bigger for all three control schemes, as can be seen from figures 13(b), 14(b), and 15(b). At this higher tracking speed, the position error profile of CCPM showed a phase lead to that by CCS. It is very interesting to note that CCPM, which stands out in high-speed, high-curvature trajectory tracking due to its speed pre-compensation, shifted the position error profile even more, in the mean time offered an obvious advantage in contour accuracy. This could be seen from figures 12(d) and
16(d); the overall contour errors at 100 mm/sec are even smaller than that at 50 mm/sec. Without this speed pre-compensation feature, the overall contour errors increased with tracking speed, as can be seen from figures 11(d) and 14(d).

Figure 16 showed comparisons of angular trajectories measured by CRS03 gyroscope on $\alpha - \beta$ plane. It is seen that pure position control system US generated trajectory with bigger infidelity. Cross-coupled compensation system could create a trajectory with satisfied fidelity and the system with speed pre-compensation (CCPM) had an edge in contour accuracy. Figure 17 showed the same comparison at tracking speed 100 mm/sec, again it is seen the trajectory could be created with schemes of cross-coupled compensation.

The conducted experiments showed that the idea proposed in this study enabled the parallel mechanism motion platform to pursue a spatial continuous trajectory. In the approach of this study, it is known that by increasing the number of control points and by increasing the number of interpolated points, accuracy could be increased to the limit of computer speed and platform hardware precision. A thorough investigation into the performance of US, CCS and CCPM with respect to speed, curvature and load duty (cutting depth) was reported in [21]. Some discussions about CCPM could be found in [27].

5 Conclusions

Parallel motion mechanisms become competing alternatives to conventional machine tools in recent years, but they are still inferior to serial machine tools in the ability of continuous trajectory tracking. Two obstacles hinder the development of trajectory tracking schemes for parallel motion mechanisms; one is the complicated forward kinematics computation in which the solution for the location of platform centroid is not unique. Another is the singular points scattered in the workspace. Some previous efforts could help parallel motion platform detour singular points and follow a spatial trajectory, but only with great trajectory inaccuracy.

A methodology that enables 3 DOF parallel motion platforms to track spatial continuous trajectory is proposed in this study. The essence is to convert the trajectory to link space and implement the proven cross-coupled contour compensation schemes. By integrating the algorithms for contour error computation and for real-time interpolation, a real-time multi-axis cross-coupled compensation structure was constructed for an experimental hydraulic parallel motion platform with Z, $\alpha$ and $\beta$ degrees of freedom. An auxiliary support helped to exclude the singularities in workspace at the expense of a slight reduction in workspace.

Experiments showed that the link-space approach could create the target spatial trajectory with adequate precision which was not possible beforehand. The proposed approach is time-efficient because complicated forward kinematics was spared; this enables a real-time interpolation in tracking of spatial continuous trajectory. The approach proposed in this study is for 3 DOF
parallel mechanisms at this stage, but a development into schemes for 6 DOF is possible.

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**Reference**


[07] Tsai, L. W., 1999, Robot analysis: the mechanics of serial and parallel manipulators, New York, John Wiley & Sons, INC.


Figure 1 The diagram of the three axes hydraulic mechanism.

Figure 2 Link-space tracking of a spatial trajectory:  \( Z = 20 \ast tt \),  \( \alpha = r(tt) \ast \sin(w(tt)) \),  \( \beta = r(tt) \ast \cos(w(tt)) \),  \( r(tt) = 0.05 \ast tt \),  \( w(tt) = \pi / 30 \ast tt \)
Figure 3 Workspace of the non-support parallel mechanism ($Z$, $\alpha$ and $\beta$).

Figure 4 Workspace of the support-reinforced parallel motion platform.
Proceedings of ICAM2007
Nov. 26-28, 2007, Tainan, Taiwan

Figure 5: The Contour error in link space

Figure 6: The path pre-compensation for a spatial curve in link space
Figure 7 Structure of the on-line, real-time interpolated cross-coupled pre-compensation control for hydraulic parallel motion platform \((Z, \alpha, \beta)\).
Figure 9 Diagram for the control of hydraulic parallel platform

Figure 10 Tracking results of uncoupled system US at 50mm/sec
Figure 11 Tracking results of cross-coupled system CCS at 50mm/sec

Figure 12 Tracking results of cross-coupled pre-compensation system CCPM at 50mm/sec
Figure 13 Tracking results of uncoupled system US at 100mm/sec

Figure 14 Tracking results of cross-coupled system CCS at 100mm/sec
Figure 15 Tracking results of cross-coupled pre-compensation system CCPM at 100mm/sec

Figure 16 Gyroscope measured trajectories on $\alpha$ - $\beta$ plane, 50 mm/sec
Figure 17 Gyroscope measured trajectories on $\alpha - \beta$ plane, 100 mm/sec