The Maximizing Convergence Rate Control for V/STOL Aircraft with Input Saturation

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Abstract—This paper presents a robust controller design for the planar vertical takeoff and landing aircraft dynamics. The controller design is based on a linearized PVTOL dynamics in terms of state and control quantities deviated from their equilibrium. The controller state-feedback gain is constructed by solving LMI conditions to maximize convergence rate while subject to the magnitude constraints on control efforts and regulated outputs. The robustness against the parasitic coupling between rolling moment and lateral force is addressed during the phase of controller design. The design results are verified and demonstrated through time-response simulations for controllers constructed according to specified quantities of the parasitic coupling.

Keywords: V/STOL aircraft, maximum convergence rate, constrained control, LMI

I. INTRODUCTION

The vertical/short takeoff and landing (V/STOL) aircraft has the capability of high mobility and maneuverability, and is especially suitable to operate in a poor runway condition such as the deck of a military vessel and an informal temporary runway. The types of V/STOL aircraft, such as the AV-8B Harrier produced by McDonnell Douglas [1] and the F-35 Joint Strike Fighter (JSF) produced by Lockheed Martin allied with Boeing [2], are among the main stream applications at present. The Harrier is powered by one Rolls Royce Pegasus F402-RR-408 turbo-fan engine with exhaust nozzles equipped in each side of the fuselage to provide the gross thrust for the aircraft. The nozzles are capable of rotating together from the aft position forward approximately 100 degrees. This change in the direction of thrust allows the aircraft to operate in the two modes of wing-borne forward flight and jet-borne hovering as well as the transition between them. In order to perform motion in lateral direction, the aircraft is equipped with reaction control valves in the nose, tails, and wingtips. By using the high pressure flow from compressor of engine, these valves produce moments around the center of mass to change attitude. On the other hand, the JSF is equipped by a modified Pratt & Whitney F119-PW-100 engine configured with shaft-driven lift fan, roll ducts and a three-bearing swivel main engine nozzle. The lift fan in the front part of frame and the main engine nozzle when swiveled vertically provide the lift force for aircraft maneuvering in a low air speed. The roll ducts in the both sides of wingtips are used for providing necessary moment for attitude handling.

In this paper, the hovering operation of the V/STOL aircraft is considered, in which the upward thrust from the main engine nozzles is manipulated by throttle for aircraft vertical motion. For attitude changing of the Harrier or JSF aircrafts, if the high pressure air from the reaction control valves or ducts induces an unexpected force component in the lateral direction, the system dynamics will exhibit non-minimum phase characteristics in case the force moves the aircraft into a lateral direction opposite to the attitude intention. Then this time delay will cause non-minimum phase effect on the aircraft dynamics due to the parasitic coupling between input moment and force.

Several non-linear controller design approaches have been conducted for this non-linear non-minimum phase planar V/STOL aircraft dynamics. One possibility is based on the well known input-output feedback linearization approach which implies a nonlinear version of pole-zero cancellation [3,4]. For a non-minimum phase system, this feedback linearization will result in internally unstable dynamics; even the linearized system is stable in the sense of input-output stability. To avoid this internal instability, feedback linearization was performed based on approximated minimum phase dynamics, in which the influence of rolling moment on the lateral force was neglected [5]. It was shown that the desired properties such as bounded tracking and asymptotic stability for the true system are maintained if the neglected parasitic coupling effect is small.

Another approach for the V/STOL aircraft control is to design a family of controllers beforehand according to the operation envelope of aircraft system. Then, in the real-time application, a mechanism for scheduling among this family of controllers is activated to realize the instantaneous controller based on the aircraft operation. In [6], based on the generic V/STOL aircraft model (GVAM), a robust design using the $\mathcal{H}_\infty$ optimal control technique was investigated. In the works of [7,8], a composite control approaches for the robust and robust gain-scheduled design were performed, respectively, for planar V/STOL aircraft dynamics. The design algorithms provided reliable solutions with guaranteed relative stability for the PVTOL aircraft possessing parasitic moment-to-force coupling effect by assuming its possible upper bound in the design algorithms. Also, the realized control law does not implemented as function of this measurable coupling parameter.

The work in [9] addressed the issue of maximizing convergence rate with input saturation for the format of a bang-bang control. The work in [10] handles the actuator saturation by enlarging the domain of attraction of the origin. This paper is aimed to provide a design robust to the parasitic coupling with maximum relative stability while subject to magnitude constraints on control effort and regulated outputs, based on given initial state conditions. The design is based on a linearized PVTOL dynamics in terms of state and control quantities deviated from their equilibrium. The controller algorithm is formulated and constructed by a suitable set of linear matrix inequality (LMI) conditions [11,12].

II. PLANAR V/STOL AIRCRAFT DYNAMICS

By restricting the aircraft to the jet-borne operation, i.e., thrust directed to the bottom of the aircraft, one can have simplified dynamics which describes the motion of the aircraft in the
vertical-lateral directions, i.e., a planar vertical/short takeoff and landing (PVTOL) aircraft. The aircraft states are the position of center of mass, \( (x, y) \), the roll angle \( \theta \), and the corresponding velocities, \( (\dot{x}, \dot{y}, \dot{\theta}) \). The control input is the thrust directed to the bottom of aircraft \( U \) and the moment around the aircraft center of mass \( U_m \). In case the bleed air from the reaction control valves or ducts produces force which is not perpendicular to the pitch axis, there will be a coupling effect between the angle rolling moment and lateral moving force. Let the ratio of lateral force induced by rolling moment be denoted by \( \varepsilon \), then the aircraft dynamics as shown in Figure 1 can be written as,

\[
\begin{align*}
-m\dddot{x} &= -\sin \theta \dot{U}_l + \varepsilon \cos \theta U_m \\
-m\dddot{y} &= \cos \theta \dot{U}_l + \varepsilon \sin \theta U_m - mg \\
J\ddot{\theta} &= U_m
\end{align*}
\]

where \( mg \) is the gravity force imposed in the aircraft center of mass and \( J \) is the moment of inertia around the axis through the aircraft center of mass and along the fuselage.

To simplify the notation of the PVTOL aircraft dynamics (1), the first and second equations in (1) are divided by \( mg \), and the third one by \( J \). Let \( x = -x/l, \ y = -y/l, \ u_l = U_l/mg, \ u_z = U_m/J, \ e = \varepsilon J/lmg \), then the normalized PVTOL aircraft dynamics (3) is obtained as,

\[
\begin{align*}
\dddot{x} &= -\sin \theta \dot{u}_l + \varepsilon \cos \theta \dot{u}_z \\
\dddot{y} &= \cos \theta \dot{u}_l + \varepsilon \sin \theta \dot{u}_z - 1 \\
\ddot{\theta} &= \dot{u}_z.
\end{align*}
\]

The term “-1” denotes the normalized gravity acceleration. The coefficient “e” denotes the parasitic coupling effect between the lateral force and rolling moment, which causes the non-minimum phase characteristic.

Let the nonlinear dynamics in (2) denoted by,

\[
\dot{x}_c = f(x_c, u),
\]

where \( x_c = (x, y, \dot{y}, \theta, \dot{\theta})^T \), \( u = (u_z, u_l)^T \). Let equilibrium state and corresponding control effort denoted as, \( x_o = (x_o, 0, y_o, 0, 0, 0)^T \), \( u_o = (1, 0)^T \). Then we can have the linearized aircraft dynamics in terms of the state and control quantities deviated from the equilibrium,

\[
\dot{\tilde{x}}_c = D_1 f(\tilde{x}_c, u_o) \tilde{x}_c + D_2 f(\tilde{x}_c, u_o) \tilde{u},
\]

where

\[
D_1 f(\tilde{x}_c, u_o) = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
D_2 f(\tilde{x}_c, u_o) = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

If the equilibrium state is considered in the origin, \( x_o = 0, y_o = 0 \), the linearized dynamics (4) can be written as,

\[
\dot{\tilde{x}}_c = D_1 f(\tilde{x}_c, u_o) \tilde{x}_c + D_2 f(\tilde{x}_c, u_o) \tilde{u}
\]

\[
:= A\tilde{x}_c + B(\varepsilon)\tilde{u} \tag{5}
\]

III. MAXIMIZING CONVERGENCE CONTROL VIA LMIS

If the static state-feedback control law, \( \bar{u} = Fx_c \), is considered for the aircraft dynamics (5). Also, the control effort is subject to actuator magnitude constraints and the standard saturation function \( \sigma(\cdot) \) is imposed on the control effort. Then the closed-loop controlled dynamics of (5) can be written as,

\[
\dot{\tilde{x}}_c = Ax_c + B(\sigma(Fx_c)) \tag{6}
\]

where \( \sigma(\bar{u}) = \text{sgn}(\bar{u}) \min \{\bar{u}_{\text{max}}, |\bar{u}| \} \) with \( \bar{u}_{\text{max}} \) the magnitude constraint of the \( i \)-th control effort.

Considering the initial state conditions, \( x_c(0) = x_o \in \mathbb{R}_{66}^d \). denote the state trajectory of system (6) as, \( \psi(t, x_o) \). Then the domain of attraction of the origin is defined as,

\[
\mathcal{L} = \{ x_o \in \mathbb{R}_{66}^d : \lim_{t \to \infty} \psi(t, x_o) = 0 \}. \tag{7}
\]

Let \( P \in \mathbb{R}_{66}^{d+} \) be a positive-definite matrix. Denote the ellipsoid defined by \( P \) as,

\[
\mathcal{E}(P, \rho) = \{ x \in \mathbb{R}_{66}^d : x^T P x \leq \rho \}. \tag{9}
\]

Let \( V(x_c) = x_c^T P x_c \). The ellipsoid \( \mathcal{E}(P, \rho) \) is said to be contractively invariant if,

\[
\dot{V}(x_c) = 2x_c^T P(Ax_c + B(\sigma(Fx_c))) < 0
\]

for all \( x_c \in \mathcal{E}(P, \rho) \setminus \{0\} \). If \( \mathcal{E}(P, \rho) \) is contractively invariant, then \( \mathcal{E}(P, \rho) \) is in the domain of attraction.

For the matrix \( F \in \mathbb{R}_{m \times n} \), define

\[
\mathcal{L}(F) = \{ x \in \mathbb{R}_{n \times 6}^d : \|F x_c\| \leq \tilde{u}_{\text{max}}, \ i \in [1, m] \}. \tag{9}
\]

Then \( \mathcal{L}(F) \) is the region in the state space where the control is admissible under the saturated state-feedback control. The contractively invariant condition in (6) can be written as,

\[
(A + BF)^T P + P(A + BF) < 0
\]

\[
\mathcal{E}(P, \rho) \subseteq \mathcal{L}(F). \tag{11}
\]

The condition in (11) can be interpreted as every state vector inside the invariant set \( \mathcal{E}(P, \rho) \) also belongs to the admissible set \( \mathcal{L}(F) \). That is, \( x_c^T P x_c \leq \rho \) leads to \( \|F x_c\| \leq \tilde{u}_{\text{max}} \), which can be expresses as the following.

\[
\begin{bmatrix}
\tilde{u}_{\text{max}}^2 \\
\|F\| \left( \frac{P}{\rho} \right)^{\frac{1}{2}}
\end{bmatrix} \geq 0. \tag{12}
\]
To guarantee that the controlled states are inside the invariant set $x_i^T \mathbf{P}_x \leq \rho$, for given initial state conditions $x_0$, it is sufficient to impose $x_i^T \mathbf{P}_x \leq \rho$, which can be written as,

$$\begin{bmatrix} 1 \\ x_i^T \\ x_0^T \end{bmatrix} \begin{bmatrix} \mathbf{P}^{-1} \\ \rho \end{bmatrix} \geq 0. \quad (13)$$

By denote $\begin{bmatrix} \mathbf{P}^{-1} \\ \rho \end{bmatrix} = \mathbf{Q}$ and $\mathbf{FQ} = \mathbf{Y}$, the matrix inequality in (10) can be written as the following LMI,

$$\mathbf{QA}^T + \mathbf{A}_q \mathbf{Q} + \mathbf{Y}^T \mathbf{B}^T + \mathbf{BY} < 0. \quad (14)$$

To maximize the convergence rate of the controlled system, (14) is modified as,

$$\mathbf{QA}_q^T + \mathbf{A}_q \mathbf{Q} + \mathbf{Y}^T \mathbf{B}^T + \mathbf{BY} < 0. \quad (15)$$

with $\beta > 0$ to be maximized. After the matrix variables $\mathbf{Q}_i$, $\mathbf{Y}_i$, are constructed, the state-feedback control law can be obtained as $\mathbf{F} = \mathbf{QY}^{-1}$. Also, (12) and (13) can be considered as the LMIs in terms of matrix variables $\mathbf{Y}_i$ and $\mathbf{Q}_i$, respectively, as following,

$$\begin{bmatrix} \mathbf{Q} \\ \mathbf{Q}_i \\ \mathbf{Y}_i \\ x_0^T \end{bmatrix} \begin{bmatrix} \mathbf{Q} \\ \mathbf{Q}_i \\ \mathbf{Y}_i \\ x_0 \end{bmatrix} \geq 0, \quad (16)$$

$$\begin{bmatrix} 1 \\ x_i^T \\ x_0^T \end{bmatrix} \begin{bmatrix} \mathbf{Q} \\ \mathbf{Q}_i \\ \mathbf{Y}_i \\ x_0 \end{bmatrix} \geq 0, \quad (17)$$

where $\mathbf{Y}_i$ is the i-row of $\mathbf{Y}$. Moreover, if the regulated output $z = \mathbf{C} x_i + \mathbf{D} \dot{u}$, (18) is considered and the magnitude constraints imposed on the regulated outputs are,

$$|z| \leq z_{i,\text{max}}, \quad (19)$$

then it can be established by

$$x_i^T (\mathbf{C}^T + \mathbf{F} \mathbf{D}^T) \mathbf{C} \mathbf{D} x_i \leq z_{i,\text{max}}^2.$$ (20)

which is equivalent to,

$$\frac{1}{z_{i,\text{max}}^2} x_i^T (\mathbf{C}^T + \mathbf{F} \mathbf{D}^T) \mathbf{C} \mathbf{D} x_i \leq x_i^T \mathbf{Q}^{-1} x_i,$$

and can be expressed as the following LMI by the Schur complement,

$$\begin{bmatrix} \mathbf{Q} \\ \mathbf{Q} \mathbf{C}^T + \mathbf{Y} \mathbf{D}^T \\ \mathbf{C} \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{Q} \\ \mathbf{Q} \mathbf{C}^T + \mathbf{Y} \mathbf{D}^T \\ \mathbf{C} \mathbf{D} \end{bmatrix} \begin{bmatrix} z_{i,\text{max}}^2 \\ z_{i,\text{max}}^2 \\ z_{i,\text{max}}^2 \end{bmatrix} \geq 0, \quad (20)$$

where $(\mathbf{C} \mathbf{D})^i$ is the j-th row of $(\mathbf{C} \mathbf{D})$.

In summary, considering the state-feedback control for the actuator constrained systems (6) with the magnitude constraints (9) on the control efforts and (19) on the regulated outputs, the LMI conditions for the maximized convergence rate design are to maximize the parameter $\beta > 0$, while subject to the LMIs in (15), (16), (17) and (20) with the matrix variables $\mathbf{Q}, \mathbf{Y}$.

### IV. SIMULATION

#### 4.1 Controller Numerical Construction

Consider the control effort capability of the Harrier aircraft dynamics used in this paper, the maximum thrust of the Pegasus engine is $\mathbf{U}_t = 10,079$ kg. The minimum operating thrust considered in the hovering operation is assumed equal to the minimum operating weight as $\mathbf{U}_t = \mathbf{m} = 6,451$ kg. Therefore, the magnitude constraint of the normalized thrust deviation can be computed as $\dot{u}_{\text{max}} = 0.6$. The magnitude constraint of the moment $\dot{u}_{2,\text{max}}$ is considered as a comparable quantity to the magnitude of thrust and chosen as $\dot{u}_{2,\text{max}} = 0.6$. The constraints on the dynamic varying parameters $\theta_{\text{max}}, \dot{\theta}_{\text{max}}$ are considered as function of the moment constraint $\dot{u}_{2,\text{max}}$, since a larger moment will tend to induce a larger amount of attitude maneuvering. The magnitude constraints are chosen as $[\dot{\theta}_{\text{max}}] = \pi \cdot \dot{u}_{\text{max}} / k$ with $k = 5$ in the design. Since the design objective is to keep tracking of the normalized position command signal in the horizontal and vertical direction, the state initial conditions in (17) are specified as $\mathbf{x}_0 = (\pm 1, 0, 0, 0, 0, 0, 0, 0)^T$ for controller construction.

By using the LMI control toolbox [13], two numerical control laws are constructed for the linearized PVTOL dynamics (5) considering the values of the parasitic coupling between the rolling moment and lateral force $\mathbf{e}$, one for $\mathbf{e} = 0$, another for $\mathbf{e} = 1$. In both cases, the maximized convergence rate is specified as $\beta = 0.2896$. Then based on the constructed matrix variables $\mathbf{Q}, \mathbf{Y}$, the resulting control law $\mathbf{F} = \mathbf{Q} \mathbf{Y}^{-1}$ can be obtained for the case of $\mathbf{e} = 0$ from,

$$\begin{bmatrix} 0 & -0.304 & 0.129 & -0.042 \\ -0.104 & -0.076 & -0.113 & -0.039 \\ 7.282 & -2.103 & 0 & -0.304 & 0.129 \\ 1.018 & -2.103 & 0 & -0.304 & 0.129 \\ 0 & -0.304 & 0.129 & -0.042 \\ 0 & -0.304 & 0.129 & -0.042 \\ 0 & -0.304 & 0.129 & -0.042 \\ 0 & -0.304 & 0.129 & -0.042 \end{bmatrix}.$$

(21a)

and for the case of $\mathbf{e} = 1$ from,

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.009 & -0.036 & 0 & 0 & -0.113 & -0.039 \\ -0.104 & -0.076 & -0.113 & -0.039 \\ 7.282 & -2.103 & 0 & 0 & -0.304 & 0.129 \\ -2.103 & 0.129 & 0 & 0 & 0.142 & 0.041 \\ -2.103 & 0.129 & 0 & 0 & 0.142 & 0.041 \\ 0 & -0.304 & 0.129 & -0.042 \\ 0 & -0.304 & 0.129 & -0.042 \end{bmatrix}.$$

(21b)

(22a)

(22b)

Referring the previous works for robust design in [7], the obtained maximum relativity stability for parasitic uncertainty $\mathbf{e} = \{0,1\}$ is $\beta' = \{0.2380,0.1815\}$ under the same control effort and varying parameter constraints. As for the gain-scheduled design in [8] under the same design condition, the
obtained maximum relativity stability for parasitic uncertainty \( \varepsilon = \{0,1\} \) is \( \beta = \{0.2504,0.2236\} \). It is true that the controller construction for the linearized dynamics (5) can achieve better performance on the relative stability than the previous works [7,8].

4.2 Time Response Simulation

In the simulation, the nonlinear normalized PVTOL dynamics (2) is controlled by the control signal \( \mathbf{u} = \mathbf{K} \mathbf{x} \). The closed-loop system is driven by reference command signal \( (x_r, y_r) \) for the lateral and vertical movement, which is generated from step command \( (x_d, y_d) \) passed through a low-pass filter with bandwidth at 10 rad/sec. With the step command \( (x_d, y_d) = (1,0),(0,1),(1,1) \), the time responses regarding the quantities of lateral-vertical position \( \{x,y\} \), attitude angle \( \{\theta, \dot{\theta}\} \), and control efforts \( \{\dot{u}_1, u_2\} \) are manifested in Figures 2-7 for the nominal design considering \( \varepsilon = 0 \) and the robust design considering \( \varepsilon = 1 \).

Figures 2-4 present the time responses for the nominal design under step tracking command in the vertical, lateral, and both vertical-lateral direction, respectively. It can be seen that the tracking performances are satisfied for these three cases. Also, the specified constraints on the control efforts, \( \dot{u}_{1,\max} \leq 0.6, u_{2,\max} \leq 0.6 \), and the output variables, \( \theta_{\max} = \pi \cdot \dot{u}_{2,\max} / 5 \), are well satisfied. Figures 5-6 show the time responses for the nominal design considering \( \varepsilon = 0 \) while operating in the nonlinear PVTOL dynamics with real-time parasitic coupling \( \varepsilon(t) = 1 \). It is shown from Figure 5 that the command following in the vertical direction is still successful. On the other hand, in the lateral command tracking as shown in Figure 6, the controlled system goes to unstable due to the undesirable coupling effect between the rolling moment and lateral force denoted by \( \varepsilon(t) \). However, if the expected quantity of the parasitic coupling \( \varepsilon = 1 \) can be considered for a robust controller design according to the linearized PVTOL dynamics (5) and the control law is obtained from the constructed matrix variables (22), the time responses in Figure 7 show that a satisfactory tracking performances can be achieved and the addressed constraints on control efforts and altitude variables can also be met.

V. CONCLUSION

This paper has presented a robust controller design for the planar vertical takeoff and landing aircraft dynamics. The controller design was based on a linearized PVTOL dynamics in terms of state and control quantities deviated from their equilibrium. The controller state-feedback gain was constructed by solving LMI conditions to maximize convergence rate while subject to the magnitude constraints on control efforts and regulated outputs. The robustness against the parasitic coupling between rolling moment and lateral force has been addressed during the phase of controller design. The design results were then verified and demonstrated through time-response simulations for controllers constructed according to specified quantities of the parasitic coupling.

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REFERENCES

Figure 2 Nominal design: vertical time response under $\epsilon(t) = 0$

Figure 3 Nominal design: lateral time response under $\epsilon(t) = 0$

Figure 4 Nominal design: lateral/vertical time response under $\epsilon(t) = 0$

Figure 5 Nominal design: vertical time response under $\epsilon(t) = 1$

Figure 6 Nominal design: lateral time response under $\epsilon(t) = 1$

Figure 7 Robust design: lateral time response under $\epsilon(t) = 1$