Effect of Market Imperfection on the Relationship between Future Index Prices and Spot Index Returns: An Empirical Study

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The degree of market imperfections affects the pricing of financial assets and the dynamic relationship among financial instruments. To investigate the dynamic interrelationship between the expected growth rate implied by the prices of index futures and the rate of return of the underlying index spot, this study examines data from the S&P 500, Nikkei 225 index futures and the TAIFEX TAIEX index futures by using the vector autoregression (VAR) model, Granger causality test, and generalized impulse response function (GIRF). The empirical result shows that the dynamic interrelationship is weaker in the mature US and Japanese markets (which represent a more perfect market) than in the emerging Taiwanese market (which represents an imperfect market). Examining the relationship between future index prices and spot index returns is an effective way of investigating market imperfections and inefficiencies.

1. Introduction

The dynamic interrelationship between futures and spot markets, has been widely discussed in the past literature. However, the pricing model of stock index futures used to be under the perfect-market hypothesis. Wang and Hsu (2006a) demonstrate that, relative to the benchmark model of perfect-market assumption, futures contracts are more likely to be priced inaccurately in a market with larger market imperfections, suggesting that the impact of market imperfections on the pricing of stock index futures is tremendous. In a subsequent paper, Wang and Hsu (2006b) empirically test the Hsu and Wang’s (2004) model and show that the model outperforms perfect-market models such as the cost of carry model and the Hemler and Longstaff (1991) model.

Wang and Hsu also recognize that it is not easy to estimate the price expectation (i.e., expected growth rate) owing to investors’ sudden change of price expectation if new information arrives. Various techniques for estimating price expectation have been employed in recent literature. One estimation method is the implied expectation, which is similar to the implied volatility in the Black-Scholes (1973) model (for example, Schmalensee and Trippi, 1978). The second estimation method is the adaptive expectation model (for example, Nerlove, 1958; Wachtel and Figlewski, 1981). However, since there is no real data for price expectation, the effectiveness of these methods is difficult to evaluate.
The price of an index future and its underlying index spot will simultaneously reflect relevant information if financial markets are perfect. Because of market imperfections, the dynamic interrelationship between index futures and spot markets exists. For instance, in the futures market in which there are lower transaction costs and fewer trading restrictions, futures prices reflect relevant information more quickly than stock prices. Moreover, the more friction the markets have, the more obvious the dynamic interrelationship between the futures price and its underlying index should be. The greater the market imperfection, the more apparent the dynamic interrelationship between index futures and spot markets. Market imperfections vary in different markets of different countries, making their dynamic interrelationship unequal. Therefore, we can examine the efficiency and maturity of a market by testing the dynamic interrelationship between futures and spot markets in that market.

Hsu and Wang’s (2004) pricing model of stock index futures in imperfect markets considers that market imperfections and incomplete arbitrage make price expectation enable to influence the price of stock index futures. Because the model can reasonably account for the pricing behavior of stock index futures, such as the phenomenon of negative bases, it has gradually attracted the attention from academic workers and practitioners. However, considering the difficulty in price expectation, we first bring the accurate estimations of dividend payouts for the USA, Japanese, and Taiwanese markets into this model to calculate the implied expected growth rate.

In this study, we investigate the dynamic interrelationship between the expected growth rate implied by the prices of index futures and the rate of return of the underlying index spot in an emerging market (Taiwan) and the developed markets (USA and Japan) by using the vector autoregression (VAR) model, Granger causality test, and generalized impulse response function (GIRF). The implied growth rate is an expected return on the underlying asset from the viewpoint of futures traders, while the rate of index return is a realized one from the viewpoint of stock traders. Theoretically, an interactive relationship between these two rates should exist. The information transmission should be faster in a developed market than in a developing market since a developed market has fewer restrictions and less market friction, which indicates the dynamic interrelationship in the U.S. and Japanese markets should be less significant than it is in Taiwan’s market. And this expectation is supported by our empirical results. Moreover, in order to examine the maturity of the financial market in Taiwan, we divide our data period into sub-period 1 and sub-period 2. The following is the importance of this paper. First, this study provides insight into how the price expectation implied by the futures prices in imperfect markets is formed. Second, this study also investigates the market efficiency by the dynamic interrelationship. Finally, this study documents evidence on the difference in the dynamic interrelationship between implied growth rates of stock index futures and rates of returns on underlying spot indices for countries with different market imperfections.

The remainder of this paper is organized as follows. Section 2 discusses related studies concerning the pricing of index futures. Section 3 describes the data and methodologies.
Section 4 reports and discusses the empirical results. The last section provides some concluding remarks.

2. Futures Pricing Model in Imperfect Markets

Because Hsu and Wang’s (2004) pricing model of stock index futures in imperfect markets is relatively new, we briefly review the model in this section.

The cost of carry model (Cornell and French, 1983) and the Hemler and Longstaff (1991) model are based on the assumption of perfect markets and no-arbitrage argument. Assuming perfect markets, a hedged position consisting of stock and futures can be “continuously” rebalanced to remain risk-free. Thus, futures valuation formulae can be obtained in perfect markets. However, sufficient reasons exist to believe that real capital markets are not perfect and index arbitrage cannot be complete. Hsu and Wang (2004) incorporate the factor of price expectation, which reflects the total effects of all market imperfections, and use an argument of the incomplete arbitrage mechanism to develop a pricing model of stock index futures in imperfect markets.

The model assumes that the stock index \( S \) follows a geometric Wiener process. Consider a hedged portfolio that consists of one unit of the underlying index and \( x \) units of the futures position. It is assumed that no initial cash outflow is required for the futures contract. Then, the rate of return of the hedged portfolio is given by

\[
\frac{dp}{p} = (w_u \sigma + u) dt + (\sigma \sigma f) dZ
\]

where \( P \) is the value of the hedged portfolio; \( w = \frac{xf}{s} \); \( F \) and \( S \) represent the prices of index futures and index spot, respectively; \( u \) and \( \sigma \) are the constant expected growth rate and the constant volatility of \( S \), respectively; \( u_f \) and \( \sigma_f \) denote the instantaneous expected return on futures, and the instantaneous standard deviation of return on futures, respectively; and \( dZ \) is a Wiener process.

If \( w = \frac{-\sigma}{\sigma_f} \), then \( w_u \sigma_f + \sigma = 0 \). In this case, the return of the hedged portfolio is certain and the portfolio is risk-free. However, to keep the portfolio risk-free, it therefore is necessary to continuously rebalance \( w_f \) until the expiration of the futures contract.

Figlewski (1989) finds that forming a risk-free hedge and rebalancing continuously until expiration is only possible in a perfect market. In imperfect markets, because arbitrage mechanism cannot be complete and index arbitrage is exposed to large risk, the portfolio cannot be risk-free at any instant. This means that the portfolio must earn some expected rate of return (which can be greater than, smaller than, or equal to the risk-free rate), rather than the risk-free rate at any instant.

Let \( u_p \) and \( \sigma_p \) denote the instantaneous expected rate of return of the hedged portfolio, and the coefficient of \( dZ \) in (1), respectively. Thus, they obtain

\[
w_f u_f + u = u_p
\]

\[
w_f \sigma_f + \sigma = \sigma_p
\]
From (2) and (3), they obtain the following partial differential equation (PDE).

$$\frac{1}{2} \sigma^2 S^2 F_{ss} + u \alpha S F_s + F_t = 0$$

(4)

where the price expectation parameter, $u = \left( (u_p - q) - \frac{\sigma_p}{\sigma} \right) \left( 1 - \frac{\sigma_p}{\sigma} \right)$, and $q$ is the dividend yield. The solution of this PDE is given by

$$F(S, t) = S_t e^{u \alpha (t - t)}$$

(5)

which is the Hsu and Wang (2004) pricing equation.

If the underlying stock index pays irregular lumpy dividends, the pricing model of stock index futures in imperfect markets can be modified as follows:

$$F(S, t) = (S_t - D_t)e^{u \alpha (t - t)}$$

(6)

where $u = \left( (u_p - u) \frac{\sigma_p}{\sigma} \right) \left( 1 - \frac{\sigma_p}{\sigma} \right)$. $D_t$ represents the present value of all cash dividends distributed by the underlying component stocks at time $t$ during the life of the futures contract. That is, $D_t = \sum_{i=1}^{n} \left( S_t, d_i, w_i, p_{i,t} \right) / e^{r(t - t)}$, where $d_i$ is the cash dividend per share for stock $i$ during the life of the futures contract; $w_i$ denotes the weight of stock $i$ in the index; $t_i$ is the time that stock $i$ pays the cash dividend; $r$ represents the risk-free interest rate; and $p_{i,t}$ is the price of stock $i$ at time $t$.

3. Methodology

3.1 Data

In this paper, we use the daily data for the S&P 500 index futures and Nikkei 225 index futures (representing the matured market) and data for the Taiwan Futures Exchange (TAIFEX) TAIEX index futures (representing the immature market) to investigate the dynamic interrelationship between the implied expected growth rates and the rates of return of their underlying spot indices, respectively. Table 1 illustrates the specifications of the TAIFEX TAIEX futures, the S&P 500 index futures contracts, and Nikkei 225 index futures.

All the nearest maturity contracts of the S&P 500, Nikkei 225 index futures and the TAIFEX TAIEX index futures have significant trading volume. To reduce thin trading problems, only the near-month contracts were considered in this paper. The trading volume of the nearby futures contract is usually large with high liquidity, and, therefore, shows the best capability of price discovery.

The sample period covers 7/21/98 to 12/30/05. To capture different market conditions for the TAIFEX TAIEX futures, two sub-samples of approximately equal length were considered. Sub-period 1 included data from 7/21/98 to 4/8/02. Sub-period 2 covered data between 4/9/02 and 12/30/05. Moreover, to reduce the asynchronous trading
problem between the index spot and the futures prices, the transaction time of each daily observation for the index futures had to approximate or equal the transaction time of each daily observation for the index spot.

Data on the S&P 500 and Nikkei 225 were collected from Datastream databases. As for the TAIFEX TAIEX futures, 30-day commercial paper rates in the secondary market were used as the proxy of risk-free interest rates. The prices of the TAIFEX futures and the TAIEX index spot, the cash dividend per share, and the commercial paper rates data are from the Taiwan Economic Journal (TEJ), AREMOS and the Taiwan Stock Exchange Corporation (TSEC).

3.2 Implied Expected Growth Rate and Index Return

In the Hsu and Wang (2004) pricing model in imperfect markets, the expected growth rate \( u^a \) is the only parameter that cannot be observed directly. They suggest that \( u^a \) can be backed out implicitly using observed futures prices. For an index futures whose underlying index pays a continuous dividend yield, such as the S&P 500 and Nikkei 225 index futures, the implied \( u^a \) at time \( t \) can be obtained from (5).

Table 1. Specifications of the TAIEX, S&P 500 and Nikkei 225 Index Futures Contracts

<table>
<thead>
<tr>
<th>Items</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: TAIEX TAIEX futures</strong></td>
<td></td>
</tr>
<tr>
<td>Underlying index</td>
<td>Taiwan capitalization weighted index (TAIEX)</td>
</tr>
<tr>
<td>Contract size</td>
<td>Futures price times NT$200</td>
</tr>
<tr>
<td>Contract months</td>
<td>Spot month, the next calendar month, and the next three quarter months</td>
</tr>
<tr>
<td>Minimum price change</td>
<td>1 index point (NT$200)</td>
</tr>
<tr>
<td>Last day of trading</td>
<td>The third Wednesday of the delivery month</td>
</tr>
<tr>
<td><strong>Panel B: S&amp;P 500 index futures</strong></td>
<td></td>
</tr>
<tr>
<td>Underlying index</td>
<td>S&amp;P 500 index</td>
</tr>
<tr>
<td>Contract size</td>
<td>Futures price times US$250</td>
</tr>
<tr>
<td>Contract months</td>
<td>March, June, September, December</td>
</tr>
<tr>
<td>Minimum price change</td>
<td>0.10 index points (US$25)</td>
</tr>
<tr>
<td>Last day of trading</td>
<td>The Thursday prior to the third Friday of the contract month</td>
</tr>
<tr>
<td><strong>Panel C: Nikkei 225 index futures</strong></td>
<td></td>
</tr>
<tr>
<td>Underlying index</td>
<td>Nikkei 225 index</td>
</tr>
<tr>
<td>Contract size</td>
<td>Futures price times ($1,000 yen)</td>
</tr>
<tr>
<td>Contract months</td>
<td>March, June, September, December</td>
</tr>
<tr>
<td>Minimum price change</td>
<td>10 index points ($10,000 yen)</td>
</tr>
<tr>
<td>Last day of trading</td>
<td>Business day preceding the 2nd Friday of each contract month</td>
</tr>
</tbody>
</table>

Source: Taiwan Futures Exchange (TAIFEX), Chicago Mercantile Exchange (CME) and Osaka Securities Exchange (OSE).
In Taiwan, because cash dividends for the underlying component stocks are generally paid once per year and are concentrated in June, the cash dividend payouts for the index are relatively lumpy. Thus, for the TAIFEX TAIEX index futures, the implied $u'_{\alpha}$ at time $t$ can be calculated from (6).

$$u'_{\alpha,t} = \frac{1}{T-t} \ln \left( \frac{F_t}{S_t - D_t} \right)$$

(7)

Following the explanation of the dividend component in (6), this study first computes the $D^t$ for (8). By utilizing (7) and (8), we derive the implied expected growth rates $u_{\alpha}$ for the US and Japanese markets and $u'_{\alpha}$ for the Taiwanese market. The rates of return on the stock index are calculated as follows:

$$R_t = \ln \left( \frac{S_t}{S_{t-1}} \right)$$

(9)

### 3.3 VAR analysis

The VAR model can be formulated as follows:

$$U_t = c_U + \sum_{i=1}^{m} \alpha_{Ui} U_{t-i} + \sum_{i=1}^{m} \beta_{Ui} R_{t-i} + \epsilon_{Ut}$$

(10a)

$$R_t = c_R + \sum_{i=1}^{m} \alpha_{Ri} U_{t-i} + \sum_{i=1}^{m} \beta_{Ri} R_{t-i} + \epsilon_{Rt}$$

(10b)

where $U_t$ denotes the expected growth rate implied by the prices of index futures and $R_t$ is the index spot return at time $t$. The number of lags ($m$) is determined by minimizing Akaike’s information criterion (AIC). The coefficients of $\alpha_{Ri}$ and $\beta_{Ui}$ measure the effects of lagged $U_t$ and lagged $R_t$ on current $R_t$ and $U_t$. If $\beta_{Ui}$ in (10a) is significant, the index spot returns lead the implied expected growth rates by $i$ periods. Similarly, if $\alpha_{Ri}$ in (10b) is significant, the implied expected growth rates transmit information $i$ periods faster than the index spot returns.

The relationship of Granger causality between $U_t$ and $R_t$ is also determined by the coefficients of $\alpha_{Ri}$ and $\beta_{Ui}$. If some coefficients of $\beta_{Ui}$ are non-zero, there exists a causality relationship from the index spot return to the implied expected growth rate. If not all coefficients of $\alpha_{Ri}$ are zero, the direction of information transformation is from the implied expected growth rate to the index spot return. A joint test of block exogeneity on $\alpha_{Ri}$ and $\beta_{Ui}$ being significantly different from zero is employed to test for the Granger causality between these two return series.

The impulse response function of the VAR model can measure the time profile of the effects of shocks on the other markets. The conventional analysis is carried out using the orthogonalized impulse responses. However, the results of traditional impulse response may differ significantly depending on the ordering of the series in the VAR system.
To avoid the problem of ordering, Koop et al. (1996) and Pesaran and Shin (1998) propose the generalized impulse response function (GIRF). The discussion of the GIRF begins with the infinite moving average representation of the VAR model given by (11).

\[ Y_t = \sum_{i=0}^{\infty} A_i e_{t-i} \]  

where \( Y_t \) is an \( nx1 \) vector of the variable under investigation; the coefficient matrices \( A_i = \Phi_1 A_{i-1} + \Phi_2 A_{i-2} + \Phi_3 A_{i-3} + \cdots + \Phi_k A_{i-k}, \ i = 1,2,\ldots \), with \( A_0 = I_n \) and \( A_i = 0 \) for \( i < 0 \); and \( e_t \) is the \( nx1 \) vector of independent identically distributed errors with mean zero and variance matrix.

Koop et al. (1996) defined the GIRF to be conditional on only one element at time \( t \), say the \( j \)th shock, and then integrated out the effects of the other shocks at time \( t \) given its value, \( e_{jt} \), where

\[ \Omega \] indicates the information set and \( h \) is the time horizon. Assuming \( E(e_{jt} | e_{jt} = e_{jt}, \Omega_{t-1}) = \Sigma s_j \sigma_{jj}^{-1} e_{jt} \), where \( \sigma_{jj} = E(e_{jt}^2) \) and \( s_j \) is a selection vector with its \( j \)th element equal to unity and zeros elsewhere. By setting \( e_{jt} = \sqrt{\sigma_{jj}} \), we obtain the effect of one unit (standard error) shock to the \( j \)th equation on the expected \( Y_{t+h} \) by

\[ \text{GIRF}_{i}(h) = \left( \frac{A_j \Sigma s_j}{\sqrt{\sigma_{jj}}} \right) \]  

4. Empirical Results

4.1 Preliminary Statistics

The expected growth rates implied by the prices of index futures (TU, SPU and NU) are a critical part of this study. Table 2 provides summary statistics for the implied expected growth rate of the TAIFEX TAIFX index futures (TU), the rate of return of TAIEX (TR), the expected growth rate in the S&P 500 futures market (SPU), and the index return of S&P 500 (SPR), the expected growth rate in the Nikkei 225 futures market (NU), and the index return of Nikkei 225 (NR).

The means and medians of NU are negative. The standard deviation of TU is twice that of SPU, and approximately four times that of UN. Figure 1 highlights the distributions of TU, SPU and NU. The skewness of NU is as high as -3.265, showing an obvious negative skew. The distribution of NU is highly leptokurtic, while the figures of TU disperse more widely than those of SPU and NU. All the null hypotheses of normal distribution are easily rejected by the test of Jarque-Bera.

Finally, the figures of Unit root test in Panel B are all significant as a return series should be, showing that there exists no co-integration between expected growth rates and index returns. Thus, using VAR to investigate the interaction between expected growth rate and index return is appropriate.
The time schedules of dividend payouts for three markets are different. While US companies pay dividends quarterly and Japanese companies pay dividends twice or thrice a year, most Taiwanese companies pay annual dividends. Estimating the annualized dividend yields in the Taiwanese market becomes a challenge. Because the dividend yields change dramatically over time, implied growth rates are often unstable.

**Figure 1. Distributions of \(TU\), \(SPU\) and \(NU\)**

![Distribution of TU](image1.png)

![Distribution of SPU](image2.png)

![Distribution of NU](image3.png)

**Table 2. Preliminary Statistics of Expected Growth Rate and Index Return**

<table>
<thead>
<tr>
<th></th>
<th>Taiwan</th>
<th>US</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(TU)</td>
<td>(TR)</td>
<td>(SPU)</td>
</tr>
<tr>
<td>Mean</td>
<td>0.043</td>
<td>0.0001</td>
<td>0.0475</td>
</tr>
<tr>
<td>Median</td>
<td>0.0325</td>
<td>-0.0005</td>
<td>0.0182</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.7994</td>
<td>0.0617</td>
<td>1.1396</td>
</tr>
<tr>
<td>Minimum</td>
<td>-1.7329</td>
<td>-0.0691</td>
<td>-0.5527</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.266</td>
<td>0.0162</td>
<td>0.1246</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.2775</td>
<td>-0.0365</td>
<td>4.0835</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>9.8271</td>
<td>4.4938</td>
<td>23.8651</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>3129.8**</td>
<td>149.2**</td>
<td>38135.2**</td>
</tr>
<tr>
<td>Observations</td>
<td>1601</td>
<td>1601</td>
<td>1823</td>
</tr>
</tbody>
</table>

**Panel B: Unit root test**

<table>
<thead>
<tr>
<th></th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(TU)</strong></td>
<td>-12.95**</td>
<td>-23.95**</td>
</tr>
<tr>
<td><strong>(SPU)</strong></td>
<td>-37.86**</td>
<td>-37.94**</td>
</tr>
<tr>
<td><strong>(NU)</strong></td>
<td>-12.94**</td>
<td>-17.07**</td>
</tr>
<tr>
<td><strong>(SRP)</strong></td>
<td>-17.64**</td>
<td>-44.63**</td>
</tr>
<tr>
<td><strong>(NR)</strong></td>
<td>-24.86**</td>
<td>-34.65**</td>
</tr>
</tbody>
</table>

\(TU\), \(SPU\) and \(NU\) represent the expected growth rate implied by index futures, and \(TR\), \(SPR\) and \(NR\) denote the rate of return of index spot for the Taiwan, US and Japan markets. Jarque-Bera statistics are for the normality test. The sample period is from 7/21/98 to 12/30/05.

** indicates significance at the 1% level.
4.2 Analysis for the Taiwan, US and Japan markets

Table 3 reports the VAR results for the Taiwanese, US and Japanese markets and analyzes the interaction between the implied growth rate and the index return. The lag order of 4 is selected by minimizing AIC.

For the Taiwanese system, there is only one significant coefficient in the TR equation. The coefficient of $TR_{t-1}$ in the $TU$ equation (-0.98) is significant, meaning that the index returns lead the implied growth rates by 1 lag. None of the $TU$ coefficients in the TR equation are significant; thus, the TR transmits information faster than does the $TU$. In the US market, all coefficients of $SPU (SPR)$ in the $SPR (SPU)$ equation are insignificant, indicating no lead-lag relationship between $SPU$ and $SPR$. As for the Japanese market, the coefficient of $NU_{t-1}$ in the $NR$ equation (0.015) is significant, meaning that the implied growth rates lead the index returns by 1 lag.

Panel B in Table 3 reports the result of the Granger causality test. Both the F-statistics (2.597 and 3.402) in the Taiwanese system are significant, indicating a bi-directional Granger causality between $TU$ and $TR$. However, combined with the previous conclusion of VAR, the causality from $TR$ to $TU$ is stronger. In U.S.A., $SPR$ and $SPU$ equations cannot be rejected, showing that neither $SPU$ nor $SPR$ Granger causes each other. In Japan, the hypotheses of $\alpha_{Rt} = 0$ in the $NR$ equations can be rejected (F-statistics=4.408), the direction of information transformation is from the implied expected growth rate to the index spot return ($NU \rightarrow NR$).

Figure 2 illustrates the result of GIRF analysis, which shows the response of implied growth rate ($U$) or index return ($R$) to within one standard deviation (S.D.) innovation of each other. Comparing Taiwanese market with Japanese and the U.S. markets, we find that the response of $R$ to one standard deviation innovation of $U$ is the strongest in Taiwanese market; following is in Japanese market. And the weakest response exists in the U.S. market. Moreover, the response period in Taiwanese market is longer than that in Japanese market, and the shortest response period is shown in the U.S. market.

On the other hand, the response of $U$ to one standard deviation innovation of $R$ is the strongest in Japanese market; following is in Taiwanese market. And the weakest response exists in the U.S. market. Moreover, the response period in Taiwanese market is longer than that in Japanese market, and the shortest response period is shown in the U.S. market.

Table 4 summarizes the dynamic interrelationship from the analyses of VAR, Granger causality, and GIRF. Both the results of VAR and Granger causality show no evidence of information transmission in the US market, while Taiwanese and Japanese markets display a significant lead-lag relationship. However, in Granger causality test, in contrast to Japan's market, Taiwan's market shows a bi-directional relationship, and it indicates that the lead-lag relationship in Taiwanese market is stronger than that in Japanese market. The GIRF analysis also demonstrates a weaker interaction in the US market, which is different from the markets of Taiwan and Japan. Summing up, the emerging market in Taiwan has a stronger lead-lag relationship than the mature US
and Japanese markets. These findings coincide with our expectations that, because of more market imperfections, the dynamic interrelationship should be more significant in a developing market.

4.3 VAR analysis for the Taiwan market in sub-periods

In the previous section, we made a comparison between an emerging market (Taiwan) and a developed market (US and Japan). In this section, we will further analyze the difference between sub-period 1 and sub-period 2 for the emerging Taiwan market. According to Wang and Hsu (2006a), the degree of market imperfection in period 1 clearly exceeds that of period 2 for the Taiwanese market. Our purpose is to see whether or not the dynamic interrelationship becomes weaker as the Taiwanese market becomes more mature.

Table 3. VAR Results for the Taiwanese, US and Japanese Markets

\[ U_t = c_U + \sum_{i=1}^{4} \alpha_{Ui} U_{t-i} + \sum_{i=1}^{4} \beta_{Ui} R_{t-i} + \varepsilon_{Ut} \]

\[ R_t = c_R + \sum_{i=1}^{4} \alpha_{Ri} U_{t-i} + \sum_{i=1}^{4} \beta_{Ri} R_{t-i} + \varepsilon_{Rt} \]

<table>
<thead>
<tr>
<th></th>
<th>Taiwan</th>
<th>US</th>
<th>Japan</th>
<th>Sub-Period 1</th>
<th>Sub-Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(TU_t)</td>
<td>(TR_t)</td>
<td>(SPU_t)</td>
<td>(SPR_t)</td>
<td>(NU_t)</td>
</tr>
<tr>
<td>Panel A: VAR model:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.0122*</td>
<td>0.0003</td>
<td>0.0112**</td>
<td>0.0001</td>
<td>-0.0058**</td>
</tr>
<tr>
<td>(U_{t1}(\alpha_1))</td>
<td>0.4916**</td>
<td>0.0027</td>
<td>0.6722**</td>
<td>0.0035</td>
<td>0.2096**</td>
</tr>
<tr>
<td>(U_{t2}(\alpha_2))</td>
<td>0.1375**</td>
<td>-0.0035</td>
<td>0.1046**</td>
<td>0.0008</td>
<td>0.0699**</td>
</tr>
<tr>
<td>(U_{t3}(\alpha_3))</td>
<td>0.0589*</td>
<td>-0.0013</td>
<td>0.0904**</td>
<td>-0.0045</td>
<td>-0.0296</td>
</tr>
<tr>
<td>(U_{t4}(\alpha_4))</td>
<td>0.0399</td>
<td>-0.0032</td>
<td>-0.1034**</td>
<td>0.0005</td>
<td>0.0399*</td>
</tr>
<tr>
<td>(R_{t1}(\alpha_1))</td>
<td>-0.9800**</td>
<td>0.044</td>
<td>0.0995</td>
<td>-0.0436</td>
<td>-0.1162</td>
</tr>
<tr>
<td>(R_{t2}(\alpha_2))</td>
<td>0.1127</td>
<td>0.0351</td>
<td>-0.0997</td>
<td>-0.037</td>
<td>0.0603</td>
</tr>
<tr>
<td>(R_{t3}(\alpha_3))</td>
<td>0.0246</td>
<td>0.0611*</td>
<td>-0.2196</td>
<td>-0.0334</td>
<td>-0.0637</td>
</tr>
<tr>
<td>(R_{t4}(\alpha_4))</td>
<td>-0.2526</td>
<td>-0.0258</td>
<td>0.2642</td>
<td>-0.019</td>
<td>0.0523</td>
</tr>
<tr>
<td>Panel B: Granger causality test</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All (\alpha_{Ri}=0)</td>
<td>3.402**</td>
<td>0.572</td>
<td>4.408**</td>
<td>2.641*</td>
<td>0.917</td>
</tr>
<tr>
<td>All (\beta_{Ui}=0)</td>
<td>2.597*</td>
<td>1.392</td>
<td>0.526</td>
<td>1.971</td>
<td>0.909</td>
</tr>
</tbody>
</table>

The figures of Granger causality test are F-statistics. ** and * indicate significance at the 1% and 5% level, respectively.
The VAR result for the Taiwanese market in two sub-periods is shown in Table 3. For sub-period 1, there is only one significant coefficient in the $TR$ equation. The coefficient of $TR_{t-1}$ in the $TU$ equation ($-1.2094$) is significant, representing that $TR$ leads $TU$ by 1 lag. Because none of the $TU$ coefficients in the $TR$ equation are significant, $TR$ plays a more important role than $TU$ in price discovery. In sub-period 2, all coefficients of $TU (TR)$ in $TR (TU)$ equation are insignificant, which indicates that there is no lead-lag relationship that period. In sub-period 1, only the $F$-value of Granger causality in the $TR$ equation (2.641) is significant, indicating that $TU$ Granger-causes $TR$. In sub-period 2 Granger-cause tests are insignificant, showing that both $TU$ and $TR$ fail to Granger-cause each other.

**Figure 2. Response to one S.D. innovation**

- **Response of $TR$ to One S.D. $TU$ Innovation**
- **Response of $TU$ to One S.D. $TR$ Innovation**
- **Response of $SPU$ to One S.D. $SPU$ Innovation**
- **Response of $SPU$ to One S.D. $SPU$ Innovation**
- **Response of $NU$ to One S.D. $NR$ Innovation**
- **Response of $NR$ to One S.D. $NU$ Innovation**
In Taiwanese market, we further compare sub-period 1 with sub-period 2. The response of \( R \) to one standard deviation innovation of \( U \) is stronger in sub-period 1 than in sub-period 2. And the response period in sub-period 1 is longer than that in sub-period 2. On the other hand, the response of \( U \) to one standard deviation innovation of \( R \) is stronger in sub-period 1 than in sub-period 2. Moreover, the response period in sub-period 1 is longer than that in sub-period 2. The responses of \( TU \) to \( TR \) innovation in sub-period 2 have a similar pattern in terms of time and magnitude. In other words, \( TU \) and \( TR \) respond to the innovation of each other faster in sub-period 2, revealing that the lead-lag relationship is stronger in sub-period 1 than in sub-period 2.

Table 4 summarizes the dynamic interrelationship from the analyses of VAR, Granger causality, and GIRF for the Taiwan market in two sub-periods. Both the results of

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Taiwan</th>
<th>US</th>
<th>Japan</th>
<th>Sub-period 1</th>
<th>Sub-period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAR</td>
<td>TR leads TU by 1 lag.</td>
<td>None</td>
<td>NU leads NR by 1 lag.</td>
<td>TR leads TU by 1 lag.</td>
<td>None</td>
</tr>
<tr>
<td>Granger causality</td>
<td>TU&lt;-&gt;TR</td>
<td>None</td>
<td>NU&lt;-&gt;NR</td>
<td>TU Granger-causes TR</td>
<td>None</td>
</tr>
<tr>
<td>GIRF</td>
<td>Stronger</td>
<td>Weaker</td>
<td>Stronger</td>
<td>Stronger</td>
<td>Weaker</td>
</tr>
</tbody>
</table>

Figure 3 GIRF for the Taiwanese market in sub-periods

Sub-period 1

![Response of TR to One S.D. TU Innovation](image1)

Sub-period 2

![Response of TR to One S.D. TU Innovation](image2)

![Response of TU to One S.D. TR Innovation](image3)

![Response of TU to One S.D. TR Innovation](image4)
VAR and Granger causality reveal no transmission evidence in period 2. The evidence of GIRF supports a weaker interaction in period 2. From the above investigation, we clearly see that the dynamic interrelationship is weaker in sub-period 1, supporting our hypothesis that the dynamic interrelationship between $TU$ and $TR$ should become weaker as the degree of market imperfection decreases.

5. Summary and Conclusions

Market imperfections influence not only the pricing of financial assets, but the dynamic relationship among financial instruments. Hsu and Wang (2004) provides a method for estimating the implied expected growth rate in their pricing model of stock index futures in imperfect markets. Because the implied rate is the price expectation of the stock index, the topic of interest in this study is the interaction between the expected and the realized returns of the index spot. This study adopts the vector autoregression (VAR) model, Granger causality test, and generalized impulse response function (GIRF) to investigate the lead-lag relationship between the expected growth rate implied by the prices of index futures and the rate of return of the underlying index spot in an emerging market (Taiwan) and a developed market (USA and Japan).

According to the results of VAR and Granger causality, no evidence of information transmission in the US market is found, while a significant lead-lag relationship is displayed in the Taiwanese market. Japan’s market is in between the US market and Taiwanese market in the degrees of lead-lag relationship. The GIRF analysis also demonstrates a weaker interaction in the US market. These findings are consistent with our expectation that the lead-lag relationship should be more significant in the emerging Taiwanese market than in the developing US market since there are fewer restrictions and less market friction in U.S.A..

This study further examines whether Taiwanese market efficiency becomes more mature with time. According to Wang and Hsu (2006a), the degree of market imperfection in the sub-period 1 is greater than that in the sub-period 2. Similarly, no transmission evidence in period 2 is found from both the results of VAR and Granger causality. And a weaker interaction in period 2 is supported by the evidence of GIRF. The above investigation coincides with our hypothesis that the more mature Taiwanese market becomes, the less significant its lead-lag relationship should be.

To summarize, the empirical result shows that the dynamic interrelationship is stronger in the Taiwanese market than in the U.S. and Japanese markets. Due to more restrictions in the Japanese market, its market imperfection is greater than that of the U.S. Nonetheless, the greater the market imperfection, the more obvious the relationship between the implied expected growth rate and the index spot return. This study provides evidence of not only the difference in dynamic pricing interrelationship between stock index futures and the underlying spot indices for markets of different imperfections, but also insights into how the price expectation implied by the price of futures in imperfect markets is formed.
Footnotes

1 Market imperfections are defined as including heterogeneous information among investors, frictional capital markets (transaction costs, taxes, etc.), constraints on short selling, and indivisibility of securities.

2 See Kawaller et al. (1987), Stoll and Whaley (1990), Chan (1992), Fleming et al. (1996), Tse (1999), and among others.

3 In Taiwan, the trading session of stocks is from 9:00 AM to 1:30 PM, and the futures market is closed fifteen minutes later than the stock market. Therefore, we collect the transaction data for both markets at 1:30 PM. Similarly, for the US markets, the trading session of stocks index futures is from 8:30 AM to 3:00 PM in mid-American time, and the S&P index futures market is closed fifteen minutes later than the stock market. Therefore, we collect the transaction data for both markets at 3:00 PM. As for Japan, its morning trading session of stocks is from 9:00 AM to 11:00 AM, and the afternoon session is from 00:30 PM to 03:00 PM. The futures market is closed ten minutes later than the stock market. Therefore, we collect the transaction data for both markets at 3:00 PM. Thus, it will solve the asynchronous trading problem between stock markets and futures markets.

4 In Taiwan, the transactions of T-bills are inactive, whereas the transactions of 30-day commercial paper are rather active and guaranteed by large banks. Therefore, the 30-day commercial paper rates in the secondary market were used as the proxy of risk-free interest rates.

References


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