Robust decentralized controller with multi-objective performance design for stochastic large-scale systems via linear matrix inequalities approach

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Abstract: The present paper focuses on a class of stochastic large-scale interconnected systems with norm-bounded and time-varying parameter uncertainties. The problem addressed is the design of a robust decentralized controller such that the corresponding interconnected closed-loop system satisfies certain multi-objective performance requirements for all admissible uncertainties. The objectives of this robust design consist of individual state variance limitation, white noise attenuation constraints on $H_2$ norm, and pole placement specification. Sufficient conditions, based on a quadratic Lyapunov function, for multi-objective robustness are derived in terms of linear matrix inequalities (LMIs). It is shown that the problem of multi-objective robust decentralized control can be solved and the effectiveness of the proposed approach is illustrated by a numerical example.

Keywords: stochastic large-scale uncertain system, robust decentralized control, multi-objective constraints, LMI

1 INTRODUCTION

Large-scale systems, consisting of a set of interconnected lower-dimension subsystems, are frequently encountered in the real world, and include power systems, digital communication networks, flexible manufacturing systems, and so on. Owing to the existence of interconnections among the subsystems, the controller design of the large-scale system is in general much more difficult than that of the individual subsystems. These difficulties motivate the development of decentralized control theory, where each subsystem is controlled independently on its locally available information. Because the advantage of this scheme in controller design is to reduce complexity and allow the control imple-
recent years, there have been great efforts in exploring the issue of decentralized stabilization for stochastic uncertain large-scale systems [8]. Some sufficient conditions for decentralized stabilization have been derived; however, it should be noted that the problem of advancing controller performance still needs to be considered.

In the study of stochastic systems, it is desired that the state variables of the closed-loop systems are maintained within a certain level of the root mean squared (r.m.s.) values. One way is to address the state covariance upper-bound condition during controller design [9]. Such an application is proposed in reference [10], where a linear quadratic Gaussian (LQG) control design problem involving a constraint on \( H_\infty \) disturbance attenuation is considered. The \( H_\infty \) constraint is embedded within the optimization process by replacing the covariance Lyapunov equation by a Riccati equation whose solution leads to an upper-bounded \( L_2 \) performance. However, no parameter uncertainty in the system matrices is taken into account in either reference [9] or reference [10], although the extended notion has been proposed elsewhere [11]. In addition, in the decoupled control scheme that is based on the property of matching condition to eliminate the interconnections for the large-scale system, generalized inverse and upper-bound covariance control (UBCC) techniques are utilized to derive a state-feedback control law such that the closed-loop states of individual subsystems can meet a given covariance upper-bound condition and the regulated outputs related to the stochastic input satisfy some \( H_\infty \) performance level [12].

It was further explored whether an additional control objective with disc-region constraint on the closed-loop poles of individual subsystems can be achieved [13]. However, the problem of the existence of interconnections in the large-scale system was not discussed in references [12] and [13]. The problem of interaction treated as disturbance has been efficiently dealt with before [14, 15]. In reference [14], it is shown that in order to achieve the sufficient conditions for overall closed-loop diagonal dominance, the interactions between the subsystems are taken as external disturbance for each isolated subsystem. Then, the methodology of the eigenstructure assignment is used to attenuate disturbances via dynamic compensators based on complete parametric eigenstructure assignment. Through attenuation of the disturbances, the closed-loop poles of the overall system are assigned in the desirable region by assigning the eigenstructure of each isolated subsystem appropriately. In reference [15], the authors deal with the problem of \( H_\infty \) control of linear two-time-scale systems which consist of slow and fast dynamic systems. Since the fast dynamics are treated as a norm-bounded disturbance such that the synthesis is performed only for the nominal slow dynamics, in the presented work all dynamics of the system, explicitly or implicitly, are taken into consideration.

Moreover, the robustly decentralized \( H_\infty \) controller (RDHC) has been proposed and developed to solve the problem of interconnection without any conservatism due to the required matching condition in references [12, 13] for the considered large-scale uncertain system. In addition, the RDHC subject to the pole placement control problem can be formulated as a convex optimization problem involving linear matrix inequalities (LMIs) such that the interconnections are also regarded as disturbance. Then, the individual subsystems can be specified with different given pole placement constraints.

The current paper concerns the design of an RDHC subject to pole placement and individual variance constraints, which together with these addressed performance conditions are so-called multi-objective constraints, for stochastic large-scale systems with parameter uncertainty. In general, a multi-objective control problem considers a mix of time- and frequency-domain specifications as presented in references [16–18], where all objectives are formulated in terms of a common Lyapunov function and controller design amounts to solving a system of LMIs. Recently, multi-objective control problems have been studied extensively in control engineering and the regulated outputs related to the stochastic input satisfy some \( H_\infty \) performance level [12].

In the present paper, attention is focused on the design of a linear decentralized state-feedback controller to satisfy multi-objective performance for stochastic large-scale systems with parameter uncertainty. The time-varying and norm-bounded parameter uncertainties appear in both state and input matrices. Based on Lyapunov stability theory and utilizing the decentralized scheme and LMI approach, a robustly decentralized stabilization for the individual subsystem in the sense of mean square asymptotically stable for all admissible uncertainties is investigated. Meanwhile, the resulting overall system is also mean square asymptotically stable.
Furthermore, with the resulting stabilization, the multi-objective design including the individual variance upper bound, disturbance attenuation level, and regional pole placement is then shown for the robust decentralized controller. Sufficient conditions for solvability of these objectives are derived in terms of LMIs. The desired state-feedback controller can be constructed through a convex optimization problem that can be efficiently handled by using standard numerical algorithms.

The paper is organized as follows. In section 2, the basic properties of stochastic large-scale uncertain systems are proposed and the desired multi-objective performance control problems are formulated. Then in section 3, based on Lyapunov stability theory, the sufficient conditions of constructing robustly stochastic stabilization are first derived in terms of LMIs. Then, an algorithm for such stabilization with satisfying multi-objective performance is also proposed. Two numerical examples are investigated in section 4 for demonstrating the effectiveness of the proposed control and showing the merits of the approach by comparison with reference [12], respectively. Finally, some conclusions are given in section 5.

## 2 PROBLEM STATEMENT AND FORMULATION

Consider a stochastic large-scale system with parameter uncertainties and linear noise entry which consists of $N$ interconnected subsystems given by the following stochastic dynamic equation

\[
\dot{x}_i(t) = (A_i + \Delta A_i)x_i(t) + (B_i + \Delta B_i)u_i(t) + D_ia(t) + W_i(t)
\]

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$, and $z_i(t) \in \mathbb{R}^{p_i}$, $i = 1, 2, \ldots, N$, are the state, control input, and controlled output of the $i$th subsystem, respectively; $w_i(t) \in \mathbb{R}^{q_i}$, $i = 1, 2, \ldots, N$, are the white noise inputs defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)$ which belong to $L_2[0, \infty)$. Furthermore, in equation (1), $A_i$, $B_i$, $D_i$, $F_i$, and $A_{ij}$, $i = 1, 2, \ldots, N, j \neq 1$, are known real constant matrices with appropriate dimensions, and $A_{ij}$ are interconnection matrices between $i$th and $j$th subsystems. $\Delta A_i(t), \Delta B_i(t), \Delta A_{ij}(t), i = 1, 2, \ldots, N, j \neq i$, are unknown matrices representing system parameter uncertainties which are assumed to be of the form

\[
\Delta A_i(t) = H_{ia} \Gamma_{ia}(t) E_{ia}
\]

\[
\Delta B_i(t) = H_{ib} \Gamma_{ib}(t) E_{ib}
\]

\[
\Delta A_{ij}(t) = H_{ij} \Gamma_{ij}(t) E_{ij}
\]

where $H_{ia}$, $H_{ib}$, $H_{ij}$, and $E_{ia}$, $E_{ib}$, and $E_{ij}$, $i, j = 1, 2, \ldots, N, j \neq i$, are known constant matrices; $\Gamma_{ia}(t)$, $\Gamma_{ib}(t)$, and $\Gamma_{ij}(t)$, $i, j = 1, 2, \ldots, N, j \neq i$, are unknown real time-varying matrix functions with Lebesgue measurable elements satisfying the following norm-bounded conditions

\[
\Gamma_{ia}(t) \leq I, \quad \Gamma_{ib}(t) \leq I, \quad \Gamma_{ij}(t) \leq I
\]

Then, $\Delta A_i(t), \Delta B_i(t)$, and $\Delta A_{ij}(t)$ are said to be admissible if both conditions (2) and (3) hold. Moreover, assume that the pair $(A_i, B_i)$ is completely stabilizable for the $i$th subsystem, $i = 1, 2, \ldots, N$, and all the state variables are measurable.

The desired linear decentralized state-feedback controller $u_i(t)$ for the $i$th stochastic uncertain subsystem (1) would be designed as

\[
u_i(t) = G_i x_i(t), \quad i = 1, 2, \ldots, N
\]

where $G_i$, $i = 1, 2, \ldots, N$, are the decentralized state-feedback gain matrices of appropriate dimensions. With the addressed controller $u_i(t)$ as shown in equation (4), the corresponding closed-loop subsystem for the $i$th subsystem (1) is given by

\[
\dot{x}_i(t) = \hat{A}_i x_i(t) + \sum_{j \neq i} \hat{A}_{ij} x_j(t) + \hat{D}_i w_i(t)
\]

where $\hat{A}_i = \tilde{A}_i + \Delta \hat{A}_i$ are composed by closed-loop nominal and uncertain matrices, that is $\tilde{A}_i = A_i + B_i G_i$ and $\Delta \hat{A}_i = \Delta A_i + \Delta B_i G_i$, respectively, and interconnected matrices $\Delta \hat{A}_i = \Delta A_i + \Delta B_i G_i$, respectively.

## 2.1 Introductory concepts and review of results

Before formulating the problem to be dealt with in this paper, some concepts of robustly stochastic stability in probability are introduced and some known results in terms of LMI conditions for the $i$th subsystem (1) are reviewed below.
2.1.1 Definition 1

The equilibrium \( x_i(t) = 0 \) of the \( i \)th nominal stochastic subsystem (1) with \( u_i(t) = 0 \) and \( w_i(t) = 0 \) is said to be mean square asymptotically stable in probability if, for any \( \varepsilon > 0 \) and initial condition \( x_i(t_0) > 0 \), there exists a \( \delta = \delta(\varepsilon) \) such that \( E[|x_i(t_0)|^2] < \delta \) is satisfied; then both probabilities \( \lim_{t \to +\infty} P[\sup_{t \geq t_0} E[|x_i(t)|^2] > \delta] = 0 \) and \( \lim_{t \to +\infty} E[|x_i(t)|^2] = 0 \) hold. Furthermore, if the \( i \)th nominal stochastic subsystem (1) is mean square asymptotically stable in probability for all admissible uncertainties as defined in conditions (2) and (3), then the subsystem (1) with \( u_i(t) = 0 \) and \( w_i(t) = 0 \) is said to be robustly stochastically stable in probability.

2.1.2 Definition 2

Consider the \( i \)th nominal stochastic subsystem (1) with \( u_i(t) = 0 \) and \( w_i(t) = 0 \). Suppose that there exists a radially unbounded, twice continuously differentiable functional \( V(x_i(t)) \) such that the expectation of the time derivative of \( V(x_i(t)) \) is negative definite, that is

\[
\dot{V}_i = E\left[ \frac{d}{dt} V(x_i(t)) \right] < 0
\]

Then, the equilibrium \( x_i(t) = 0 \) of the subsystem (1) is mean square asymptotically stable in probability. Moreover, it is also robustly stochastically stable in probability if the condition exists for subsystem (1) with all admissible uncertainties.

2.1.3 Definition 3

The subsystem (1) with \( w_i(t) = 0 \) is said to be robustly stochastically stabilizable in probability via state feedback if there exists a linear decentralized state-feedback control law (4) such that the resulting overall interconnected stochastic closed-loop system (5) satisfies the sense of Definition 2 for all admissible uncertainties.

2.1.4 Definition 4

A subset \( D \) of the complex plane is called an LMI region if there exist a symmetric matrix \( L \) and a matrix \( M \) such that

\[
D = \{ s \in C : f_D(s) < 0 \}
\]

with

\[
f_D(s) = L + sM + \bar{s}M^T
\]

where \( C \) denotes the set of complex numbers and \( \bar{s} \) is the complex conjugate of \( s \).

2.1.5 Definition 5

Let \( D \) be a given LMI region. A dynamic system \( x(t) = Ax(t) \) is called \( D \)-stable if all of its poles lie in \( D \) (i.e. all eigenvalues of the matrix \( A \) lie in \( D \)).

2.1.6 Lemma 1

The reader is referred to reference [12]. Let \( D \) be a given LMI region as described in equation (7). The dynamic system with matrix \( A \) is \( D \)-stable if, and only if, there exists a symmetric and positive definite matrix \( Q \) such that

\[
R_D(A, Q) < 0
\]

where

\[
R_D(A, Q) = L \otimes Q + M \otimes (AQ) + M^T \otimes (AQ)^T
\]

In equation (10), the notation \( \otimes \) denotes the Kronecker product of matrices.

2.1.7 Remark 1

The feasibility of pole region (7) is equal to the matrix inequality (9) which was proven in reference [12] and as a counterpart of Gutman’s theorem for LMI regions. The results relating \( f_D(s) \) in equation (8) to \( R_D(A, Q) \) in equation (10) are obtained by the substitution \( (1, s, \bar{s}) \leftrightarrow (Q, AQ, QA^T) \).

2.2 Formulation of the problem

Now, the purpose of the current paper is to design a linear decentralized state-feedback control law (4) for the interconnected stochastic uncertain system (1) such that the corresponding closed-loop system (5) is robustly stochastically stable in probability and is also able to achieve the following proposed multi-objective performance for all admissible uncertainties. Each one of them can be formulated as follows.

2.2.1 Objective (i): constraints on the \( H_\infty \) norm

The desired \( H_\infty \) performance level is described as [11]

\[
\|H_i(s)\|_\infty = \sup_{w_i(t)} \left[ \frac{E[|z_i(t)|^2]}{\|w_i(t)\|_2} \right] \leq \gamma_i
\]

with

\[
w_i(t) \in L_2(0, \infty), \quad i = 1, 2, \ldots, N
\]

where \( H_i(s) \) denotes the closed-loop transfer function from \( w_i(t) \) to \( z_i(t) \) for the system (5) and the
performance-level upper bound $\gamma_i$ is a positive scalar which can be implemented as a constraint to be met or a parameter to be minimized during the controller construction.

2.2.2 Objective (ii): constraints on the pole placement region

Considering the region of the disc $D(-q_i, \rho_i)$ with centre at $(-q_i, 0)$ and radius $0 < \rho_i < q_i$ for the pole of the $i$th closed-loop system as the given LMI region, the matrix parameters of equation (9) can be found as

$$L = \begin{bmatrix} -\rho_i & q_i \\ q_i & -\rho_i \end{bmatrix}, \quad M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$  \hspace{1cm} (12)

Under the decentralized scheme, the $i$th closed-loop system (5) can be considered in the situation of interconnected states $x_i(t) = 0$. Then, equation (5) can be represented as

$$\dot{x}_i(t) = \tilde{A}_i x_i(t) + D_i w_i(t)$$

$$z_i(t) = F_i x_i(t), \quad i = 1, 2, \ldots, N$$  \hspace{1cm} (13)

By Definition 5, the subsystem (13) is said to be $D$-stable if all the eigenvalues of the closed-loop system matrix $\tilde{A}_i$ lie within the LMI disc region $D(-q_i, \rho_i)$, that is

$$\lambda(\tilde{A}_i) \in D(-q_i, \rho_i)$$  \hspace{1cm} (14)

2.2.3 Objective (iii): constraints on the state covariance upper bound

The individual steady-state variance of the system (5) satisfies the following constraint

$$[\tilde{X}_i]_{kk} = \text{var}(x_{ik}(t)) \leq [X_i]_{kk} \leq (\sigma_i^2), \quad k = 1, 2, \ldots, n_i$$  \hspace{1cm} (15)

where $\text{var}(x_{ik}(t))$ and $\sigma_i^2$, respectively, denote the $k$th variance value and r.m.s. constraints for the variance of the $i$th subsystem; $[X_i]_{kk}$ denotes the $k$th diagonal element of the upper-bound covariance matrix $X_i$; $[\tilde{X}_i]_{kk}$ denotes the $k$th diagonal element of the covariance matrix $\tilde{X}_i$ which can be defined as follows

$$\tilde{X}_i = \lim_{t \to \infty} E[x_i(t)x_i^T(t)]$$  \hspace{1cm} (16)

3 CONTROLLER DESIGN

3.1 Introduction

In this section, based on Lyapunov stability theory, the sufficient conditions of constructing the robustly stochastic stabilization are first derived in terms of LMIs. Then, such stabilization which achieves the performance addressed in Objectives (i) to (iii) is also proposed. Before proceeding further, some lemmas are given which are useful in the proof of this work.

3.1.1 Lemma 2

The reader is referred to reference [22]. Let the symmetric matrix $S$ be partitioned as

$$S = \begin{bmatrix} Y & \Phi \\ \Phi^T & Z \end{bmatrix}$$  \hspace{1cm} (17)

with $Y$ and $Z$ being symmetric matrices. By Schur Complement, $S$ is positive definite if, and only if, either

$$\begin{cases} Z > 0 \\ Y - \Phi Z^{-1} \Phi^T > 0 \end{cases}$$  \hspace{1cm} (18)

or

$$\begin{cases} Y > 0 \\ Z - \Phi^T Y^{-1} \Phi > 0 \end{cases}$$  \hspace{1cm} (19)

3.1.2 Lemma 3

The reader is referred to reference [23]. Let $U, V, W$, and $\Gamma(t)$ be real matrices of appropriate dimensions, with $\Gamma(t)$ satisfying the norm-bounded condition $\Gamma(t)\Gamma^T(t) \leq I$, $\forall t$. Then we have:

(a) for any matrix $\hat{Q} > 0$ and scalar $\alpha > 0$

$$\hat{Q}(U\Gamma(t)V) + (U\Gamma(t)V)^T \hat{Q} \leq \alpha Q U U^T \hat{Q} + \alpha^{-1} V^T V$$  \hspace{1cm} (20)

(b) for any matrix $\hat{Q} > 0$ and scalar $\alpha > 0$ such that $\hat{Q} - \alpha U U^T > 0$

$$(W + U\Gamma(t)V)\hat{Q}^{-1}(W + U\Gamma(t)V)^T \leq W^T (\hat{Q} - \alpha U U^T)^{-1} W + \alpha^{-1} V^T V$$  \hspace{1cm} (21)

3.2 Robustness of the decentralized stabilization

For the overall stochastic closed-loop system, the augmented form of dynamic subsystem (5) can be represented as

$$\dot{x}(t) = A_c x(t) + D_c w(t)$$

$$z(t) = F_c x(t)$$  \hspace{1cm} (22)
where
\[ x(t) = [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T \]
\[ w(t) = [w_1^T(t), w_2^T(t), \ldots, w_N^T(t)]^T \]
\[ z(t) = [z_1^T(t), z_2^T(t), \ldots, z_N^T(t)]^T \]
\[ D_e = \text{diag} \{ D_i \} \]
\[ F_e = \text{diag} \{ F_i \}, \quad i = 1, 2, \ldots, N \]
\[ A_e = \begin{bmatrix} \hat{A}_1 & \hat{A}_1 & \cdots & \hat{A}_{1N} \\ \hat{A}_{21} & \hat{A}_2 & \cdots & \hat{A}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{A}_{N1} & \hat{A}_{N2} & \cdots & \hat{A}_N \end{bmatrix} \]

(23)

Now define the following quadratic Lyapunov function for the system (22)
\[ V(x(t)) = x^T(t)Px(t) \]
(24)

where \( x(t) \) is as previously defined and \( P = \text{diag} \{ P_i \} \) with \( P_i = P_i^T > 0, \quad i = 1, 2, \ldots, N \). Substituting the dynamic system (22) into the expectation of time derivative of the quadratic Lyapunov function (24) gives
\[
E \left[ \frac{d}{dt} V(x(t)) \right] = \sum_{i=1}^{N} E \left[ \frac{d}{dt} (x_i^T(t)P_i x_i(t)) \right] = \sum_{i=1}^{N} E \left[ \frac{d}{dt} (x_i(t)) \right] \]
\[
\sum_{i=1}^{N} E \left[ x_i^T(t)(\hat{A}_i P_i + P_i \hat{A}_i) + (\Delta \hat{A}_i P_i + P_i \Delta \hat{A}_i) x_i(t) \right] + x_i^T(t)P_i D_e w_i(t) + w_i^T(t)D_i^T P_i x_i(t) + \sum_{j \neq i}^{N} (x_i^T(t)P_i A_{ij} + \Delta A_{ij}) x_j(t) + x_i^T(t)(A_{ij} + \Delta A_{ij})^T P_i x_i(t) \right] \]
\[
= \sum_{i=1}^{N} \tilde{V}_i \] (25)

where \( V(x_i(t)) = x_i^T(t)P_i x_i(t) \) and \( \tilde{V}_i \) is as defined in equation (6). From equation (2) and Lemma 3
\[
x_i^T(t) (\Delta \hat{A}_i P_i + P_i \Delta \hat{A}_i) x_i(t) \leq x_i^T(t) (\alpha_{ia} P_i H_a H_a^T P_i + \alpha_{ia} E_{ia}^T E_{ia}) + \alpha_{ib} P_i H_b H_b^T P_i + \alpha_{ib} (E_{ib} G_i)^T (E_{ib} G_i) x_i(t) \]
(26)

and
\[
x_i^T(t) P_i \Delta A_{ij} x_j(t) + x_j^T(t) (\Delta \hat{A}_j P_i + P_i \Delta \hat{A}_j) x_i(t) \leq \alpha_{ij} x_i^T(t) P_i H_{ij} H_{ij}^T P_i x_i(t) + \alpha_{ij}^{-1} x_j^T(t) E_{ij}^T E_{ij} x_j(t) \]
(27)

where \( x_{ia}, x_{ib}, \) and \( x_{ij}, i = 1, 2, \ldots, N, \) are positive numbers to be determined by the designer. In addition, for any symmetric and positive matrix variables \( R_{ij}, i = 1, 2, \ldots, N, \) \( j \neq i, \) the following expression is always true
\[
x_i^T(t) P_i A_{ij} x_j(t) + x_j^T(t) A_{ij}^T P_i x_i(t) \leq x_i^T(t) P_i A_{ij} R_{ij} A_{ij}^T P_i x_i(t) + x_j^T(t) R_{ij}^{-1} x_j(t) \]
(28)

Let \( w_i(t) \equiv 0 \) and following from equations (25) to (28), then, by Definition 3, the sufficient condition of robustly stochastic stability for the overall closed-loop system (22) can be obtained as
\[
\sum_{i=1}^{N} \tilde{V}_i \leq \sum_{i=1}^{N} E [x_i^T(t)J_i x_i(t)] < 0 \]
(29)

which implies that
\[
J_i = (A_i + B_i G_i)^T P_i + P_i (A_i + B_i G_i) + \alpha_{ia} P_i H_a H_a^T P_i + \alpha_{ib}^{-1} E_{ia}^T E_{ia} + \alpha_{ib} P_i H_b H_b^T P_i + \alpha_{ib}^{-1} (E_{ib} G_i)^T (E_{ib} G_i) + \sum_{j \neq i}^{N} (P_i A_{ij} R_{ij} A_{ij}^T P_i + R_{ij}^{-1} + \alpha_{ij} P_i H_{ij} H_{ij}^T P_i) \]
(30)

Pre- and post-multiplying \( X_i = P_i^{-1} \) for both sides of equation (30) yields
\[
X_i(A_i + B_i G_i)^T + (A_i + B_i G_i) X_i + \alpha_{ia} H_a H_a^T X_i + \alpha_{ib}^{-1} X_i E_{ia}^T E_{ia} X_i + \alpha_{ib} H_b H_b^T X_i + \alpha_{ib}^{-1} (E_{ib} G_i)^T (E_{ib} G_i) X_i + \sum_{j \neq i}^{N} (A_{ij} R_{ij} A_{ij}^T X_i + R_{ij}^{-1} X_i + \alpha_{ij} H_{ij} H_{ij}^T X_i) \]
(31)

By applying Schur Complement to equation (31) and using \( L_i = G_i X_i, \) the robustly stochastic decentralized stabilization problem for the stochastic large-scale interconnected closed-loop system (5) is summarized as the following proposition.

3.2.1 Proposition 1

Consider the stochastic large-scale uncertain system (1) with \( w_i(t) \equiv 0 \) satisfying the assumption (2). Then the system is robustly stochastically stabilizable in
probability via state feedback for all admissible uncertainties if there exist some matrices $X_i = X_i^T > 0$, $R_i = R_i^T > 0$, $L_i$, and some positive real numbers $\alpha_{ii}$, $\alpha_{ib}$, and $\alpha_{ji}$, $i, j = 1, 2, \ldots, N$, $j \neq i$, such that the LMI condition

$$
\begin{bmatrix}
\Theta_i & X_i E_i^T & L_i^T E_i^T & X_i \bar{E}_i^T & \bar{X}_i \\
E_{ii} X_i & -\alpha_{ii} I & 0 & 0 & 0 \\
E_{ib} X_i & 0 & -\alpha_{ib} I & 0 & 0 \\
\bar{E}_i X_i & 0 & 0 & -Y_{ii} & 0 \\
\bar{X}_i & 0 & 0 & 0 & -\Pi_{ii}
\end{bmatrix} < 0
$$

(32)

is satisfied, where

$$
\Theta_i = X_i A_i^T + A_i X_i + B_i L_i + L_i^T B_i^T + \bar{A}_i \Pi_{ii} \bar{A}_i^T + \alpha_{ii} H_i A_i + \alpha_{ib} H_i b_i + \bar{H}_i Y_i \bar{H}_i^T
$$

$$
\bar{X}_i = [X_i, \ldots, X_i], \quad \bar{E}_i = [E_{ii}, \ldots, E_{ii}]^T
$$

$$
\bar{A}_i = [A_i, \ldots, A_{ii}], \quad \bar{H}_i = [H_i, \ldots, H_{ii}]
$$

$$
\Pi_{ii} = \text{diag}(R_{ii}, \ldots, R_{ii}), \quad \Pi_{ii} = \text{diag}(R_{ii}, \ldots, R_{ii})
$$

$$
Y_i = \text{diag}(\alpha_{ii}, \ldots, \alpha_{ii}), \quad Y_i = \text{diag}(\alpha_{ii}, \ldots, \alpha_{ii})
$$

$$
r, l, i = 1, 2, \ldots, N, \quad r, l \neq i
$$

Moreover, a decentralized stabilizing feedback control law is given by $u_i(t) = G_i x_i(t)$ with $G_i = L_i X_i^{-1}$.

### 3.3 Robustness of the decentralized stabilization with $H_\infty$ norm constraints

In the current literature, there are many ways to eliminate the effects of the external disturbance. One of them is the $H_\infty$ technique. Indeed, in section 3.2, the resulting robustly stochastic stabilization is not capable of rejecting exogenous disturbance. To achieve Objective (ii), this is equivalent to enhancing the resulted stabilization with $H_\infty$ performance. The following proposition is provided a solution by using the $H_\infty$ approach to solve the problem of robustly stochastic stabilization with $H_\infty$ norm constraints.

#### 3.3.1 Proposition 2

Consider the stochastic large-scale uncertain system

$$
X_i = X_i^T > 0, \quad R_i = R_i^T > 0, \quad L_i, \quad \text{and some positive real numbers } \alpha_{ii}, \alpha_{ib}, \text{ and } \alpha_{ji}, \quad i, j = 1, 2, \ldots, N, \quad j \neq i,
$$

such that the LMI condition

$$
\begin{bmatrix}
\Theta_i & X_i E_i^T & L_i^T E_i^T & X_i \bar{E}_i^T & \bar{X}_i \\
E_{ii} X_i & -\alpha_{ii} I & 0 & 0 & 0 \\
E_{ib} X_i & 0 & -\alpha_{ib} I & 0 & 0 \\
\bar{E}_i X_i & 0 & 0 & -Y_{ii} & 0 \\
\bar{X}_i & 0 & 0 & 0 & -\Pi_{ii}
\end{bmatrix} < 0
$$

(34)

is satisfied, where $\Theta_i$, $\Pi_{ii}$, $Y_{ii}$, $\bar{E}_i$, and $\bar{X}_i$ are as defined previously in equation (33). Moreover, a decentralized stabilizing feedback control law is given by $u_i(t) = G_i x_i(t)$ with $G_i = L_i X_i^{-1}$.

#### 3.3.2 Proof

The $H_\infty$ performance constraint of the overall closed-loop system (5) can be obtained from the individual $H_\infty$ norm performance $E[\|x(t)\|_2] < \gamma_1 \|w_i(t)\|_2$ for all non-zero $w_i(t) \in L_2[0, \infty)$ as

$$
\sum_{i=1}^{N} \int_{0}^{\infty} (\gamma_i^2 t z_i(t) - \gamma_i^2 w_i(t)) dt < 0
$$

(35)

Now, we define

$$
\mathcal{H}(t) = \sum_{i=1}^{N} \left\{ E \left[ \int_{0}^{\infty} (\gamma_i t z_i(t) - \gamma_i^2 w_i(t)) dt \right] + \int_{0}^{\infty} \tilde{V}_i dt \right\} - E[V(x(\infty))] - V(x(0))
$$

(36)

where

$$
E[V(x(\infty))] - V(x(0)) = \sum_{i=1}^{N} \int_{0}^{\infty} E \left[ \frac{d}{dt} V(x_i(t)) \right] dt = \sum_{i=1}^{N} \int_{0}^{\infty} \tilde{V}_i dt
$$

(37)

Subject to the zero initial condition $x_i(t) = 0$ for all $t \leq 0$ and noting that $V(x(t)) > 0$

$$
\mathcal{H}(t) \leq \sum_{i=1}^{N} \left\{ E \left[ \int_{0}^{\infty} (\gamma_i t z_i(t) - \gamma_i^2 w_i(t)) dt \right] + \int_{0}^{\infty} \tilde{V}_i dt \right\}
$$

(38)
Substituting the expression of $\dot{V}_i$ as defined in equation (25) into equation (38) yields
\[
\sum_{i=1}^{N} \mathbb{E} \left\{ \int_{0}^{\infty} \left[ x_i(t) (J_i + F_i^T F_i) x(t) + x_i(t)^T P_i D_i w_i(t) + w_i(t)^T D_i^T P_i x(t) - \gamma_i^2 w_i(t)^T w_i(t) \right] dt \right\}
\]
\[
= \sum_{i=1}^{N} \mathbb{E} \left\{ \int_{0}^{\infty} \left[ x_i(t)^T \left[ J_i + F_i^T F_i \right] x_i(t) - \gamma_i^2 w_i(t)^T w_i(t) \right] dt \right\}
\]
\[
\times \left[ x_i(t)^T \right] \right\} \right) \right\}
\]
\[
< 0 \quad (39)
\]
where $J_i$ is defined as previously in equation (30). By letting $\kappa \to \infty$ and combining the condition in equation (35), the following quadratic inequality can be derived
\[
\sum_{i=1}^{N} \mathbb{E} \left\{ \int_{0}^{\infty} \left[ x_i(t)^T \left[ J_i + F_i^T F_i \right] x_i(t) - \gamma_i^2 I \right] \left[ w_i(t)^T \right] \right\} \times < 0 \quad (40)
\]
By Schur Complement, the quadratic inequality condition (40) is equivalent to
\[
\left( A_i + B_i C_i \right)^T P_i + P_i \left( A_i + B_i C_i \right) + \alpha_{ia} P_i L_i H_i^T H_i P_i
\]
\[
+ \alpha_{ia}^{-1} E_{ia}^T E_{ia} + \alpha_{ia} P_i L_i H_i^T H_i P_i + \alpha_{ia}^{-1} \left( E_{ia}^T G_i \right)^T \left( E_{ia}^T G_i \right)
\]
\[
+ \sum_{j=1}^{N} (P_i A_j R_{ij} A_j^T P_i + R_{ji}^{-1})
\]
\[
+ \alpha_{ij} P_i H_i^T H_j P_i + \alpha_{ij}^{-1} E_{ij}^T E_{ij}
\]
\[
+ F_i^T F_i + \gamma_i^2 P_i D_i D_i^T P_i < 0 \quad (41)
\]
Pre- and post-multiplying $X_i = P_i^{-1}$ for both sides of equation (41) and using $L_i = G_i X_i$, then the proof is completed by re-applying Schur Complement to equation (41).

3.4 Robustness of the decentralized stabilization with pole placement constraints

In this section, the problem of the closed-loop system with poles required to lie in a given LMI disc region $D(-q_i, r_i)$ which is contained in the left-half plane for the $i$th individual subsystem is dealt with in the following proposition.

3.4.1 Proposition 3

Consider the subsystem (1) with the interconnected states $x_i(t) = 0$, $j = 1, 2, \ldots, N$, $j \neq i$, that satisfies assumption (2). Let $D(-q_i, r_i)$ be a given disc LMI region in complex $s$-plane for the $i$th subsystem. Then the matrix $\hat{A}_i$ is called $D$-stable via the state feedback for all admissible uncertainties if, and only if, there exist a positive definite matrix $X_i$, matrix $L_i$, and some real positive numbers $\beta_{ipa}$, $\beta_{ipb}$, $\delta_{ipa}$, and $\delta_{ipb}$, $l = 1, 2, \ldots, N$, such that the LMI condition
\[
\begin{bmatrix}
\Psi_i & X_i A_i^T + L_i B_i^T \zeta_i^T & \zeta_i^T \\
A_i X_i + B_i L_i - q_i (X_i - \tilde{\zeta}_{ipa} H_i L_i - \tilde{\zeta}_{ipb} H_i) & 0 & 0 \\
\zeta_i & 0 & -\delta_i \delta_i^T
\end{bmatrix}
\]
\[
< 0
\]
(42)
is satisfied, where
\[
\Psi_i = X_i A_i^T + A_i X_i + B_i L_i + L_i B_i^T + \beta_{ipa} H_i L_i^T H_i + \beta_{ipb} H_i H_i^T + \delta_{ipa} \delta_{ipb} \quad (43)
\]
Moreover, a decentralized stabilizing feedback control law is given by $u_i(t) = G_i x_i(t)$ with $G_i = L_i X_i^{-1}$.

3.4.2 Proof

Considering the $D$-stable property of system (5) and according to the Lemma 1
\[
R_{dp}(\hat{A}_i, P_i) = L \otimes P_i + M \otimes P_i \hat{A}_i + M^T \otimes \hat{A}_i P_i < 0
\]
(44)
where $P_i = P_i^T > 0$ and the matrix parameters $L$ and $M$ are given in equation (12). Substitution of equation (12) into equation (44) yields
\[
R_{dp}(\hat{A}_i, P_i) = \begin{bmatrix}
-\rho_i P_i & q_i P_i + P_i \hat{A}_i \\
q_i P_i + \hat{A}_i P_i & -\rho_i P_i \end{bmatrix} < 0
\]
(45)
By Schur Complement, equation (45) can be rewritten as
\[
-\rho_i P_i + \{q_i P_i + P_i \hat{A}_i\} (\rho_i P_i)^{-1} \{q_i P_i + \hat{A}_i P_i\} < 0
\]
(46)
According to assumption (2)
\[
\Delta A_i + \Delta B_i G_i = \begin{bmatrix}
H_{ia} & H_{ib}
\end{bmatrix} \begin{bmatrix}
\Gamma_{ia}(t) & 0 \\
0 & \Gamma_{ib}(t)
\end{bmatrix} \begin{bmatrix}
E_{ia} \\
E_{ib}
\end{bmatrix}
\]
(47)
where $H = [H_{ia} \ H_{ib}]$, $E = [E_{ia}^T \ (E_{ib} G_i)^T]^T$, and $\Gamma(t) = \text{diag} [\Gamma_{ia}(t) \ \Gamma_{ib}(t)]$. Substituting equation (47) into
equation (46) yields
\[ P_{i}((\dot{A}_{i} + H(t)E) + (\dot{A}_{i} + H(t)E)^{T}P_{i} \]
\[ + q_{i}^{-1} \{ (q_{i}^{2} - \rho_{i}^{2})P_{i} \}
\[ + (\dot{A}_{i} + H(t)E)P_{i}(\dot{A}_{i} + H(t)E)^{T} \} \]
\[ < 0 \]  
(48)

By Lemma and some positive real numbers \( \beta_{1,pa}, \beta_{1,pb}, \)
\( \xi_{ipa}, \) and \( \xi_{ipb}, \)
\[ P_{i}(H(t)E) + (H(t)E)^{T}P_{i} \]
\[ \leq \beta_{1,pa}P_{i}H_{ia}H_{ia}^{T}P_{i} + \beta_{1,pa}E_{ia}E_{ia} \]
\[ + \xi_{ipa}E_{ia}E_{ia} + \xi_{ipb}(E_{ib}G_{i})^{T}(E_{ib}G_{i}) \]  
(49)

and
\[ (\dot{A}_{i} + H(t)E)P_{i}(\dot{A}_{i} + H(t)E)^{T} \]
\[ \leq \tilde{\Lambda}_{i}(P_{i}^{-1} - (\xi_{ipa}H_{ia}H_{ia}^{T} + \xi_{ipb}H_{ia}H_{ia}^{T})^{-1})\dot{A}_{i} \]
\[ + \xi_{ipa}E_{ia}E_{ia} + \xi_{ipb}(E_{ib}G_{i})^{T}(E_{ib}G_{i}) \]  
(50)

With the aid of equations (49) and (50), the inequality (48) can be represented as
\[ P_{i}(\dot{A}_{i} + \dot{H}(t)E)P_{i} + \beta_{1,pa}P_{i}H_{ia}H_{ia}^{T}P_{i} + \beta_{1,pa}E_{ia}E_{ia} \]
\[ + \beta_{1,pa}P_{i}H_{ia}H_{ia}^{T}P_{i} + \beta_{1,pa}(E_{ib}G_{i})^{T}(E_{ib}G_{i}) \]
\[ + q_{i}^{-1} \{ (q_{i}^{2} - \rho_{i}^{2})P_{i} \}
\[ + (\dot{A}_{i} + H(t)E)P_{i}(\dot{A}_{i} + H(t)E)^{T} \} \]
\[ < 0 \]  
(51)

Pre- and post-multiplying \( X_{i} = P_{i}^{-1} \) for both sides of equation (51) and using \( L_{i} = G_{i}X_{i} \), the inequality (42) is obtained by application of Schur Complement.

3.5 Constraints on the upper bound of state covariance

The following proposition gives a solution for solving the problem of the constraints on the upper bound of state covariance.

3.5.1 Proposition 4

Consider the desired upper bound on the state covariance of the system (5) as described in equation (15). Let \( (\sigma_{k}^{2})_{i} > 0, \) \( k = 1, 2, \ldots, N, \) \( i = 1, 2, \ldots, n_{i}, \) be a set of given real positive constants. If there exists the positive definite matrix \( X_{i} \) such that the LMI condition
\[ \begin{bmatrix} -(\sigma_{k}^{2})_{i} I_{n_{i}} & X_{i} \end{bmatrix} \begin{bmatrix} P_{i} & \end{bmatrix}^{T} < 0, \]
\[ X_{i}X_{i}^{T} - X_{i} \]
(52)

is satisfied, where the row vector \( l_{k} \in R^{n_{i} \times n_{i}} \) with the kth element equal to 1 and others equal to zero, then the upper bound of state covariance constraint can be achieved.

3.5.2 Proof

Rewriting equation (15), one has
\[ I_{n_{i}}X_{i}l_{k}^{T} \preceq \sigma_{k}^{2}_{i}, \]
\[ k = 1, 2, \ldots, n_{i}, \ i = 1, 2, \ldots, N \]  
(53)

or equivalently
\[ -(\sigma_{k}^{2})_{i} + l_{i}X_{i}(X_{i}^{-1})X_{i}l_{k}^{T} \preceq 0 \]  
(54)

Using the property of Schur Complement, the inequality (54) can be reformulated in the LMI form as shown in equation (52).

Now, the above derivation for multi-objective performance can be summarized into the following main theorem.

3.5.3 Main theorem

Consider system (1) that satisfies the assumption (2), given \( \chi_{i} > 0, q_{i} > 0, \rho_{i} > 0, \) and \( (\sigma_{k}^{2})_{i} > 0, \) \( k = 1, 2, \ldots, n_{i}, \)
\( i = 1, 2, \ldots, N. \) Then the system (1) is robustly stochastically stabilizable in probability, with Objectives (i) to (iii) held via state feedback for all admissible uncertainties, if there exist some matrices \( X_{i} = X_{i}^{T} > 0, \)
\( R_{i} = R_{i}^{T} > 0, \)
\( L_{i}, \) and some positive real numbers \( \chi_{ia}, \)
\( \chi_{ib}, \beta_{ipa}, \beta_{ipb}, \xi_{ipa}, \xi_{ipb}, \) \( i = 1, 2, \ldots, N, \) \( j \neq i, \)

satisfying the LMI conditions (34), (42), and (52).

3.5.4 Proof

Following the proofs of Propositions 2 to 4, one knows that the multi-objective performance constraints (i) to (iii) can be achieved by the suitable convex optimization problem as shown in LMI conditions (34), (42), and (52). In other words, if matrices \( X_{i} \) and \( L_{i} \) exist and satisfy these LMIs, then the control feedback gain achieves the multi-objective performance constraints (i) to (iii) and can be synthesized by \( G_{i} = L_{i}X_{i}^{-1}. \)

3.5.5 Remark 2

In this paper, the problem of the RDHC design for stochastic large-scale uncertain systems is investigated. The issue of the interconnection \( \sum_{j=1}^{N} A_{ij}x_{j}(t) \) is addressed in Proposition 2 for controller design. If we further impose the pole placement and individual variance constraints, the design requirements can be formulated as a convex optimization problem. Following the concepts of reference [10], the exogenous disturbance can be ignored in formulating the pole placement specification. Therefore, once the interconnection of the
system is considered as external disturbance, it can be ignored. This assumption may cause some conservatism in Proposition 3. However, the feasibility of the convex optimization problem, which contains the interconnections in the RDHC design, can guarantee that the poles of the closed-loop system lie in a stable disc region.

4 NUMERICAL EXAMPLES

Two numerical examples are investigated in this section for demonstrating the effectiveness of the proposed control and showing the merits of the approach by comparing the result with reference [12].

4.1 Example 1

Consider the stochastic uncertain system consisting of two subsystems as follows:

(a) the first subsystem

\[
\dot{x}_1(t) = (A_1 + \Delta A_1)x_1(t) + (B_1 + \Delta B_1)u_1(t) + (A_{12} + \Delta A_{12})x_2(t) + D_1w_1(t)
\]

\[
z_1(t) = F_1x_1(t)
\] (55)

(b) the second subsystem

\[
\dot{x}_2(t) = (A_2 + \Delta A_2)x_2(t) + (B_2 + \Delta B_2)u_2(t) + (A_{21} + \Delta A_{21})x_1(t) + D_2w_2(t)
\]

\[
z_2(t) = F_2x_2(t)
\] (56)

The states \(x_1(t) = [x_{11}(t) \ x_{12}(t)]^T\), \(x_2(t) = [x_{21}(t) \ x_{22}(t)]^T\), and the system matrices are

\[
A_1 = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & 0 \\ 0.2 & 1 \end{bmatrix}
\]

\[
D_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad F_1 = [1 \ 0], \quad \Delta A_1 = \begin{bmatrix} r_1(t) & 0 \\ 0 & 0 \end{bmatrix}
\]

\[
\Delta B_1 = \begin{bmatrix} v_1(t) \\ 0 \end{bmatrix}, \quad \Delta A_{12} = \begin{bmatrix} 0 & \mu_{12}(t) \\ 0 & 0 \end{bmatrix}
\]

\[
A_2 = \begin{bmatrix} 0 & 1 \\ -4 & 5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0 & 0.3 \\ 0 & 0 \end{bmatrix}
\]

\[
D_2 = \begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix}, \quad F_2 = [1 \ 0], \quad \Delta A_2 = \begin{bmatrix} r_2(t) & 0 \\ 0 & 0 \end{bmatrix}
\]

\[
\Delta B_2 = \begin{bmatrix} v_2(t) \\ 0 \end{bmatrix}, \quad \Delta A_{21} = \begin{bmatrix} 0 & 0 \\ \mu_{21}(t) & 0 \end{bmatrix}
\]

where \(|r_i(t)| \leq 1\), \(|v_i(t)| \leq 1\), and \(|\mu_{ij}(t)| \leq 1\), \(i, j = 1, 2\), \(j \neq i\) respectively.

The desired control objectives according to equations (11), (14), and (15) are specified as follows.

1. Objective (i): the \(H_\infty\) norm constraints are

\[
\|H_1(s)\|_\infty \leq \gamma_1 \text{min} \quad \|H_2(s)\|_\infty \leq \gamma_2 \text{min}
\] (57)

where \(\gamma_1 \text{min}\) and \(\gamma_2 \text{min}\) are parameters to be minimized during design.

2. Objective (ii): the pole placement region constraints are

\[
\lambda(\hat{A}_1) \in D(-q_1, \rho_1) = D(-25, 22)
\]

\[
\lambda(\hat{A}_2) \in D(-q_2, \rho_2) = D(-20, 18)
\] (58)

3. Objective (iii): the state covariance upper-bound constraints are

\[
\text{var}(x_{11}) \leq (\sigma_1^2) = 0.5, \quad \text{var}(x_{12}) \leq (\sigma_2^2)_1 = 1
\]

\[
\text{var}(x_{21}) \leq (\sigma_1^2)_2 = 1.5, \quad \text{var}(x_{22}) \leq (\sigma_2^2)_2 = 0.8
\] (59)

Suppose that \(x_1(0) = [1, 2]^T\) and \(x_2(0) = [5 \ 8]^T\), then the proposed multi-objective design can be carried out as follows.

1. Step 1: from assumption (2), the various known matrices are

\[
H_{1a} = H_{1b} = H_{2a} = H_{2b} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}
\]

\[
H_{21} = H_{1b} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_{1a} = E_{2a} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

\[
E_{1b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E_{2b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad E_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}
\]

\[
E_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \Gamma_{ia}(t) = \text{diag} \{r_i(t)\}
\]

\[
\Gamma_{ib}(t) = \text{diag} \{v_i(t)\}, \quad \Gamma_{ij}(t) = \text{diag} \{\mu_{ij}(t)\}
\]

\(i, j = 1, 2, \quad j \neq i\)

2. Step 2: the state feedback gain matrices \(G_1\) and \(G_2\) that achieve the above design specifications (57), (58), and (59) for the closed-loop systems of equations (55) and (56) can be constructed by using the generalized eigenvalue minimization problem (GEVP) method in the MATLAB LMI control toolbox, subject to the LMI conditions.
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(34), (42), and (52), as

\[ G_1 = \begin{bmatrix} -40.0716 & -15.9253 \end{bmatrix} \]

and

\[ G_2 = \begin{bmatrix} -18.0148 & 34.7150 \end{bmatrix} \]

with performance minimization \((\gamma_1)_{\text{min}} = 0.3061\) and \((\gamma_2)_{\text{min}} = 0.9016\)

3. Step 3: the complete control laws for each subsystem are then

\[
u_1(t) = \begin{bmatrix} -40.0716 & -15.9253 \end{bmatrix} x_1(t) \]

\[
u_2(t) = \begin{bmatrix} -18.0148 & 34.7150 \end{bmatrix} x_2(t) \]

Substitution of the control laws (61) and (62) into the corresponding subsystem (55) and (56) gives the Fig. 2

The second poles location of \(\lambda(\tilde{A}_1)\) for the second subsystem

variances of states \(\text{var}(x_{11}) = 0.1817, \text{var}(x_{12}) = 0.9016, \text{var}(x_{21}) = 1.2467,\) and \(\text{var}(x_{22}) = 0.7061,\) respectively, satisfy the individual variance constraint given in equation (59).

4.2 Example 2

The reader is referred to reference [12]. A linear time-invariant stochastic system is written in the form of two subsystems

\[
\dot{x}_1(t) = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(t) + \delta \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} x_2(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w_1(t)
\]

\[
z_1(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x_1(t)
\]

and

\[
\dot{x}_2(t) = \begin{bmatrix} 0 & 1 \\ -4 & 5 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u_2(t) + 
\]

\[
+ \delta \begin{bmatrix} 0 & 0 \\ 3 & 3 \end{bmatrix} x_1(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} w_2(t)
\]

\[
z_2(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x_2(t)
\]

where the states \(x_1(t)\) and \(x_2(t)\) are as defined previously and \(\delta \in [-1, 1]\). Now, the design purpose is to find a state-feedback controller for satisfying the control objectives \(\|H_1(p)\|_{\infty} \leq 1\) and \(\|H_2(p)\|_{\infty} \leq 0.8\), and \(\text{var}(x_{11}(t)) < 2.5, \text{var}(x_{12}(t)) < 3, \text{var}(x_{21}(t)) < 1,\)
Fig. 3 The first poles location of $\lambda(\tilde{A}_2)$ for the second subsystem

Fig. 4 The second poles location of $\lambda(\tilde{A}_2)$ for the second subsystem

and $\text{var}(x_{22}(t)) < 2$, which are the same control objectives as in reference [12], with the new proposed approach. Table 1 presents the results obtained from this approach and reference [12]. It should be pointed out that the control objectives of this example are concerned with $H_\infty$ norm constraint and variance

Table 1 Example 2: comparison with reference [12]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value by using reference [12]</th>
<th>Value by using approach in this work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{var}(x_{11})$</td>
<td>$0.3564$</td>
<td>$0.2359$</td>
</tr>
<tr>
<td>$\text{var}(x_{12})$</td>
<td>$0.8490$</td>
<td>$0.3791$</td>
</tr>
<tr>
<td>$\text{var}(x_{21})$</td>
<td>$0.3055$</td>
<td>$0.3334$</td>
</tr>
<tr>
<td>$\text{var}(x_{22})$</td>
<td>$0.7068$</td>
<td>$0.6131$</td>
</tr>
<tr>
<td>$G_1$</td>
<td>$[-1.1522 - 4.7043]$</td>
<td>$[-11.9003 - 16.3499]$</td>
</tr>
<tr>
<td>$G_2$</td>
<td>$[-10.3754 - 12.9754]$</td>
<td>$[-235.9482 - 209.2502]$</td>
</tr>
</tbody>
</table>

constraint combined. Thus this problem can be regarded as a suboptimal control problem and the feasibility problem (FEASP) method can be utilized instead of the GEVP method used in Example 1.

Referring to Table 1, some conclusions can be drawn in comparing the new approach with reference [12], as follows.

1. As shown in Table 1, all the variance values of states used by the proposed approach are less than those in reference [12], except $\text{var}(x_{21})$. Although the state-feedback gains used by the present approach are larger, the controller used herein can satisfy the addressed objectives without any conservatism regarding the match conditions as found in reference [12].
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multi-objective performance for the stochastic large-scale system with time-varying and norm-bounded parameter uncertainties. It has been shown that the robust decentralized state-feedback controller for a large-scale interconnected stochastic uncertain system can be designed if a set of linear matrix inequalities is solvable. Ultimately, the effectiveness of the proposed approach has been shown in Example 1 and comparison is made with reference [12] in Example 2. In further study, the results of this paper will be explored in relation to some high-performance complex systems.

REFERENCES


Fig. 7 The time response for the first subsystem with the control law $u_1(t)$

Fig. 8 The time response for the second subsystem with the control law $u_2(t)$

2. In order to facilitate the comparison with reference [12], in the current example, the controller designed by the proposed approach has not incorporated the pole placement constraint. Therefore, the merits of the present approach, which include the decreased settling time and the efficient upgrading of the state-feedback gain, are hidden. However, with respect to the requirement for different performances, the present approach shows better design flexibility.

5 CONCLUSION

The present paper has studied the problem of robustly stochastic decentralized stabilization with...


APPENDIX

Notations

A > B is positive definite

A–B is positive definite

diag(·) diagonal matrix of {·}

E[·] expectation operator

I identity matrix with appropriate dimensions

L2(0, ∞) space of square-integrable vector functions over [0, ∞)

Rn identity matrix with appropriate dimensions

Rn×m -dimensional Euclidean space

the set of n × m dimensional real matrix

(·)T transpose of the vector or matrix (·)

∥·∥ Euclidean vector norm

L2(0, ∞) norm

∥·∥2 norm

λmin(A) minimum eigenvalue of matrix A

(Ω, F, (Ft)1<∈R+∗, P) a complete probability space with a filtration (Ft)1<∈R+∗