THE OPTIMIZATION OF SPECTRUM IN THE POWER ANALYZER

Rong-Ching Wu*, Hsiao-Ming Chen*, Ting-Chia Ou**, and Jong-Ian Tsai***

*Department of Electrical Engineering, I-Shou University, Kaohsiung, Taiwan, R.O.C
**Department of Electrical Engineering, National Sun Yat-Sen University, Kaohsiung, Taiwan, R.O.C
***Department of Electronic Engineering, Kao Yuan University, Kaohsiung, Taiwan, R.O.C.

ABSTRACT
The purpose of this paper is to promote the analysis accuracy of the spectrum in the power analyzer. The accuracy of the spectrum is most decided by the sampling rate. Different sampling rates cause their relative sum of amplitudes. This paper also advances the V curve searching method to find the optimal sampling rate. Because the relation between the sum of amplitude and the sampling rate determines the character of the V curve, the optimal solution will be found quickly.

KEY WORDS
optimization, sampling rate, power analyzer.

1. Introduction

Power system measurements, including magnetic field measurement, non-linear load measurement, and harmonic measurement etc., digital signal processing always plays an important role [1-3]. The application of harmonic measurement includes amplitude, phase, real power, reactive power, apparent power, and equivalent impedance of each harmonic and total harmonic distortion. According to the different purposes, the business instrument of harmonic measurement can be divided into [4-6]:

(a) Spectrum analyzer: It can analyze integral and non-integral harmonics.
(b) Harmonic analyzer: It can analyze the magnitudes of harmonics.
(c) Distortion analyzer: It can show total harmonic distortion.
(d) Digital harmonic measuring equipment: It uses digital filtering and FFT to quickly and largely catch the measured signal. It can enable the PC to immediately analyze signals.

From the spectrum, the different frequency components of a time domain signal can be separated. The spectrum can display important information which might hide in the data, process complex systems, and get the parameter quickly. Discrete Fourier transformation (DFT) is usually adapted as the tool to find the spectrum [7]. After issue of the FFT, this raises the computing velocity of the spectrum and fully generates the analysis capability to achieve rejuvenation. FFT plays an important role in harmonic analysis, and nowadays FFT is the most popular tool in spectrum analysis.

Owing to the limitations of application in FFT, errors happen from misapplication. The most familiar errors are the picket-fence effect and leakage effect [8]. They result from the sampling period being not equal to the time of signal period.

This paper establishes a complete method to solve the above-mentioned defect. By adjusting the sampling rate, the frequency scale can be altered to match the signal parameter, the leakage effect can be eliminated, and the exact parameter will be displayed on the spectrum.

The picket-fence effect is caused by the harmonic frequency being unable to match the spectral lines. Under the condition of precise analysis, the signal characteristics cannot be changed to match with the spectral lines - the only way is changing the spectral line to match the signal characteristics.

The change of spectral line can be achieved by changing the sampling rate or sampling number. When a meter is used for measuring a 60 Hz power system, the sampling number is generally sat \( N = 128 \), and the sampling rate is sat \( R = 128 \times 60 \). Two problems occur when changing the spectral line by the sampling number. First, if the sampling number is not equal to \( 2^r \), the FFT cannot be used and DFT takes the transform algorithm instead; this will increase computation significantly. Secondly, under the above-mentioned condition, the alteration of each sampling number will cause the alteration of frequency scale about 0.5 Hz; this value is much larger than the frequency variation of the power system under normal operation. Therefore, the actual method is changing the spectral line by the sampling rate.

There are two problems with re-sampling the signal. If the original signal wasn’t saved in a digital system, it must be ascertained that the signal characteristic is the same as it was before. The condition can hardly be achieved in the actual measurement. If the original is saved in a digital system, the data are discrete. The numerical method must be used when getting the new data from the discrete signal. This sampling method will be responsible for more or less error, but it can assure that the characteristic of the re-sampling signal is the same as it was before. This paper re-samples the digital signal and uses Lagrange interpolation to solve the problem of interpolation.

The method of optimal spectrum has been proposed [9,10], however, for accurate calculation in the interpolated FFT of this method, the frequency difference between each two harmonics must be larger than six spectral lines. In power analyzers, the sampling time is nearly a period of the signal; it will make the spectral lines of adjacent order
2. Theory
2.1 The largest variation of amplitude summation

This paper hypothesizes that the signal of power systems has the following characteristics: (1) the signal is periodic; (2) harmonics are integral; and (3) following frequency increase, harmonics are feeble. As the signal is periodic, harmonics become discrete on the spectrum, and when the energy centralizes on the identical spectral line, analysis will be optimal, but because harmonics are feeble following frequency increasing, the distortion will be less after re-sampling. Because harmonics are integral, the optimal spectrum will be achieved by changing the sampling rate.

First, we discuss the condition of optimization. If a signal consists of a cosine component, the signal can be expressed as

\[ x(n) = A \cos(2\pi f_1 n + \phi_1) \quad n = 0, 2, ..., N - 1 \]  

where \( A \) are the amplitudes, \( \phi_1 \) are the phases, and \( f_1 \) are the frequency. When this signal is transformed into the spectrum, the relationship between the highest amplitude and signal is

\[ A_p = \frac{NA}{2\pi} \sin \frac{\pi \delta}{\pi \delta} \]  

where \( \delta \) is the frequency bias. Because of the effect of leakage, the energy leaks to the adjacent spectral lines. If a spectral line away from the peak is \( m' \) and its amplitude is \( A_{p+m'} \), the relation between the energy \( A_{p+m'} \) and \( A_p \) is

\[ \frac{A_{p+m'}}{A_p} = \frac{\delta}{\delta + m'} \]  

And according to Cauchy-Schwarz inequality

\[ (\sum_A_m)^2 \geq \sum A^2_m \]  

For a periodic signal, the right-hand side of the function (4) is the signal energy.

\[ \sum A^2_m = A^2 \]  

The signal energy is constant, so the left-hand side of function (5) can obtain a minimum value. Because the energy of the periodic signal has a centripetal characteristic, we make the energy center from \( p - q \) to \( p + q \). Then the amplitude summation is

\[ AS = \sum A_m \frac{NA}{2\pi} \sum \sin \frac{\delta}{\delta - m'} \]  

From (6), the minimum amplitude summation appears in \( \delta = 0 \).

Another factor to effect the amplitude summation is Parseval’s Theorem. With different sampling rates, the energies of signals are different. This fact makes the optimal sampling rate be located at the local minimum of the amplitude summation but not the global minimum. Fig. 1 shows the amplitude summation corresponding to different sampling rates. The amplitude summation decreases following the sampling rate, which is the effect of Parseval’s Theorem. Besides, the optimal solution, \( R = 7603.2 \), is located at the local minimum. This condition is not the global minimum, but is the largest variation in this region. For this reason, this paper takes the largest variation of amplitude summation to be the characteristic of the optimal sampling rate.

2.2 Optimal solution searching

In Fig. 1, the relation between amplitude summation and the sampling rate is near the type of V curve. The optimal solution is in the lowest point. This lowest point can be found by three known conditions. From Fig. 2, the three known conditions are \((x_1, y_1), (x_2, y_2), (x_3, y_3)\). The V curve method regards the relationship among them is a symmetrical V curve, that is

\[ (y^* - y_2)(x_2 - x_1) = (y_2 - y_1)(x^* - x_2) \]  

or

\[ (y_2 - y_3)(x_3 - x_1) = (y_3 - y_1)(x_2 - x_3) \]  

By function (9) to (10), the estimated optimal solution can be derived as

\[ x^* = \left[ x_2 - y_2 \right] x_1 - x_2 \]  

or

\[ y^* = \left[ y_2 - y_1 \right] x_1 - x_2 \]  

Thus, the amplitude summation

\[ AS = \sum A_m \frac{NA}{2\pi} \sum \sin \frac{\delta}{\delta + m'} \]  

We make the energy center from \( p - q \) to \( p + q \). Then the amplitude summation is

\[ AS = \sum A_m \frac{NA}{2\pi} \sum \sin \frac{\delta}{\delta + m'} \]  

From (6), the minimum amplitude summation appears in \( \delta = 0 \).

Another factor to effect the amplitude summation is Parseval’s Theorem. With different sampling rates, the energies of signals are different. This fact makes the optimal sampling rate be located at the local minimum of the amplitude summation but not the global minimum. Fig. 1 shows the amplitude summation corresponding to different sampling rates. The amplitude summation decreases following the sampling rate, which is the effect of Parseval’s Theorem. Besides, the optimal solution, \( R = 7603.2 \), is located at the local minimum. This condition is not the global minimum, but is the largest variation in this region. For this reason, this paper takes the largest variation of amplitude summation to be the characteristic of the optimal sampling rate.
This $y^*$ can be used to decide whether (11) or (12) is the equation to find the estimated optimal solution. Via the new $x^*$, the new amplitude summation can be obtained. However, the relation is not a completely symmetrical V curve. It must through run through iteration to obtain the optimal solution. The three known conditions are taken to calculate the lowest point. Since the new value has been obtained, it can replace the most divergent condition, and be a new reference data.

2.3 Re-sampling

This paper uses Lagrange’s interpolation method to process the re-sampling work. When the new time series is $n'$, it relates to the coordinates of the original signal which can make reference to contiguous $l+1$ information which is $x(n_0)$, $x(n_1)$, ..., and $x(n_l)$. The new value can be derived as

$$x(n') = \sum_{i=0}^{l} \frac{1}{\prod_{j=0}^{j=n_i - n_j}} x(n_i)$$

Equation (13) is the $l$ order Lagrange’s interpolation method. When $l$ is two, the three contiguous data are taken as the reference. When $l$ is four, the five contiguous data are taken as the reference. The rest may be deduced by analogy. The work of re-sampling will be solved.

A signal consists of harmonics in different frequencies. The smoother the harmonic, the more the characteristic will be reserved after re-sampling. The frequencies of smooth harmonics are always low. Therefore, the re-sampling of low frequency harmonics can preserve the characteristic as like the original. The steeper the harmonic, the less the characteristic will be preserved after re-sampling. The frequencies of steep harmonics are always high. Therefore, the re-sampling of high frequency harmonics will distort the characteristic from the original. The relation between the distortion and frequency ratio is shown in Fig. 3. When the frequency of harmonic is lower than 1/4 of the sampling rate, there is a very good result from re-sampling. This paper suggests the sampling rate is higher than 4 times the highest frequency harmonic, and then the re-sampling signal will be close to the original one.

3. Procedure

This section rearranges the above theory as a complete procedure.

Step 1: signal sampling: Decide the sampling period $T$ and sampling rate $R$. In this step, the sampling period must be appropriate and enable the spectrum of each band to be discriminated. The sampling period must be appropriate and ensure avoidance of any aliasing effect.

Step 2: reference signals: Re-sample the original signal by (13) with a higher and a lower sampling rate. Then, including the original signal, the three reference signals are established.

Step 3: time-frequency transformation: Signals are transformed into spectra by FFT.

Step 4: amplitude summation: The amplitude summation sums all amplitudes on the spectrum.

Step 5: optimal solution searching: The lowest point of V curve can be calculated and judged by (11) and (12). Then, the estimated optimal sampling rate has been found.

Step 6: convergence examining: If the results have converged to the accepted range, the procedure stops taking iterations and ends the program.

Step 7: re-sampling: Re-sampling the original signal by the estimated optimal sampling rate.
Step 8: time-frequency transformation: The new signals are transformed into spectra by FFT.
Step 9: amplitude summation: This step is the same as step 4.
Step 10: renewing reference signals: The new signal replaces the most divergent condition, and becomes a new reference data.
Step 11: go to step 5.

4. Ability evaluation

This section evaluates the ability of this method in three parts. The first illustrates the results of re-sampling. The second discusses the convergence. The last shows the comparison of different methods.

4.1 Results of re-sampling

In order to explain the procedure of this method, this paper uses a standard measured signal in power electronics as an example. The first five harmonics of the current in the three-phase six-pulse commutator can be expressed as

\[
x(t) = 4.6587 \cos(2\pi \cdot 59.9t - 2.0065) \\
+ 0.891 \cos(2\pi \cdot 299.5t - 0.858) \\
+ 0.290 \cos(2\pi \cdot 419.3t - 0.845) \\
+ 0.110 \cos(2\pi \cdot 658.9t - 1.160) \\
+ 0.076 \cos(2\pi \cdot 778.7t - 1.119)
\]  

(14)

The sampling number is \( N = 128 \), and the sampling rate is \( R = 7680 \) s/sec. The waveform is shown as Fig. 4. Many zero-crossing points appear in this waveform. Generally, power instruments refer to zero-crossing points to calculate the period of signal. However, in the case of the signal being complex or disturbed by noise, the zero-crossing method will cause errors. This method can effectively avoid the disadvantage of the zero-crossing method. After re-sampling the original signal with the different sampling rate, the reading values of the spectrum are shown in Table 1. It can be found that the reading values will be altered to follow the sampling rate. The second and third harmonics must be zero. For the leakage effect, the spectral lines, which would be zero, will cause errors. In Table 1, it can be found that a better result is located at the condition \( R = 7664.64 \). Fig 5 also shows that the local minimum of amplitude summation appears near the condition \( R = 7664.64 \).

4.2 Convergence

This section refers the \((7603.2, 7680, 7756.8)\) of the stated example to run the optimal solution search. Each iterative result is shown in Table 2. It can be found that an accurate result is achieved after the first iteration. Owing to the amplitude summation having a characteristic of the V curve, the optimal solution can be found quickly. In this example, the optimal sampling rate is \( R = 7664.28 \), and the amplitude summation is 12.08. The optimal solution must be located among three reference data. When analyzing the signal in the 60 Hz power systems, the initial reference data could be sat \((59.5, 60.0, 60.5)\).
Table 2

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Ref. 1</th>
<th>Ref. 2</th>
<th>Ref. 3</th>
<th>Optimum</th>
<th>AS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7603.2</td>
<td>7680</td>
<td>7756.8</td>
<td>7659.355</td>
<td>12.333</td>
</tr>
<tr>
<td>2</td>
<td>7603.2</td>
<td>7659.355</td>
<td>7680</td>
<td>7664.280</td>
<td>12.157</td>
</tr>
<tr>
<td>3</td>
<td>7659.355</td>
<td>7664.280</td>
<td>7680</td>
<td>7664.206</td>
<td>12.080</td>
</tr>
<tr>
<td>4</td>
<td>7659.355</td>
<td>7664.206</td>
<td>7664.280</td>
<td>7664.279</td>
<td>12.082</td>
</tr>
<tr>
<td>5</td>
<td>7664.206</td>
<td>7664.279</td>
<td>7664.280</td>
<td>7664.280</td>
<td>12.080</td>
</tr>
<tr>
<td>6</td>
<td>7664.279</td>
<td>7664.280</td>
<td>7664.280</td>
<td>7664.280</td>
<td>12.080</td>
</tr>
</tbody>
</table>

4.3 Comparison of different methods

This subheading compares results of this method and traditional FFT as in Table 3. There are some errors with frequency, amplitude, and phase caused by FFT, but the spectrum will achieve accurate results by this method. In the analysis process, this method must carry on three FFT and search by V curve. Less time is required for these steps, so the method can be used for real-time analysis.

5. Conclusion

The purpose of this paper is to promote the accuracy of the spectrum in the power analyzer. The accuracy of the spectrum is affected by different sampling rates. This study evaluates analysis results of spectra under different sampling rates by amplitude summation, and purpose V curve search to find the optimal sampling rate. As soon as the optimal sampling rate is found, the optimal spectrum is found as well. Owing to the relationship between the amplitude summation and the sampling rate being a type of V curve, the optimal solution can be obtained quickly. The paper also compares results of FFT and the optimal spectrum to prove the accuracy of this method.

Acknowledgement

The authors thank the project of NSC 96 - 2622 - E - 214 - 011 - CC3, which supports the research of this paper.

References


Table 3

<table>
<thead>
<tr>
<th>Harmonic Parameter</th>
<th>Real</th>
<th>FFT</th>
<th>This method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>59.9</td>
<td>60/0.17</td>
<td>59.877/0.04</td>
</tr>
<tr>
<td>Amplitude</td>
<td>4.659</td>
<td>4.661/0.04</td>
<td>4.657/0.04</td>
</tr>
<tr>
<td>Phase</td>
<td>-2.007</td>
<td>-2.012/0.16</td>
<td>-2.005/0.06</td>
</tr>
<tr>
<td>5th</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>299.5</td>
<td>300/0.17</td>
<td>299.386/0.04</td>
</tr>
<tr>
<td>Amplitude</td>
<td>0.891</td>
<td>0.892/0.11</td>
<td>0.888/0.33</td>
</tr>
<tr>
<td>Phase</td>
<td>-0.859</td>
<td>-0.887/0.89</td>
<td>-0.851/0.25</td>
</tr>
<tr>
<td>7th</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>419.3</td>
<td>420/0.17</td>
<td>419.140/0.04</td>
</tr>
<tr>
<td>Amplitude</td>
<td>0.290</td>
<td>0.295/1.72</td>
<td>0.287/1.03</td>
</tr>
<tr>
<td>Phase</td>
<td>-0.846</td>
<td>-0.887/1.3</td>
<td>-0.837/0.29</td>
</tr>
<tr>
<td>11th</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>658.9</td>
<td>660/0.17</td>
<td>658.649/0.04</td>
</tr>
<tr>
<td>Amplitude</td>
<td>0.110</td>
<td>0.113/2.73</td>
<td>0.108/1.82</td>
</tr>
<tr>
<td>Phase</td>
<td>-1.160</td>
<td>-1.162/0.06</td>
<td>-1.094/2.1</td>
</tr>
<tr>
<td>13th</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>778.7</td>
<td>780/0.17</td>
<td>778.403/0.04</td>
</tr>
<tr>
<td>Amplitude</td>
<td>0.076</td>
<td>0.080/5.26</td>
<td>0.074/2.63</td>
</tr>
<tr>
<td>Phase</td>
<td>-1.119</td>
<td>-1.183/2.04</td>
<td>-1.111/0.25</td>
</tr>
</tbody>
</table>

Note: reading value/percentage error (%)