Flow Behavior of Polyacrylamide Solution. III: Mathematical Treatment

Mu-Hoe Yang
Department of Chemical Engineering, Kao Yuan Institute of Technology
Kaohsiung County, 82101 Taiwan, Republic of China

ABSTRACT

The flow behavior of polyacrylamide solutions was systematically determined over a wide range of temperatures (20-50 ℃) and concentrations (20-50ppm), using a coaxial cylinder viscometer. The results indicated the rheological behavior of low concentration polyacrylamide solution behaves as non-Newtonian fluids at all these concentrations. The effect of temperature on the consistency coefficient and flow behavior index of polyacrylamide solution of the different concentrations followed an Arrhenius-type relationship. Moreover, the effect of concentration on consistency coefficient and flow behavior index followed exponential-law relationship at the temperatures used. The rheological constants for the Arrhenius and exponential-law models were determined. The combined effect of temperature and concentration on the coefficient of dynamic shear stress can be represented by a single equation:

\[ \text{shear stress} = 2.446 \times 10^{-7} \exp(0.0639C + 3613/RT) \cdot \exp(0.00707C - 245/RT) \cdot (\text{shear rate})^{2.337}. \]

INTRODUCTION

The polyacrylamide solution is important for using coagulation of wastewater treatment process. Hence, the rheological behavior of polyacrylamide solution is very important in solution properties and engineering calculations related to the handling of the polymer solution for addition in wastewater treatment[1-4]. The effect of
various components of polyacrylamide solution on its flow behavior under low shear rate was recently reported [5]. In the case of polyacrylamide solution, the rheological behavior of polyacrylamide homopolymer solution in water depends not only on shear rate, but also on such variable, as concentration and temperature. The effect of various medium concentration polyacrylamide solutions on its flow behavior was recently reported [6]. It is unknown whether the flow behavior of polyacrylamide solution conforms to power law for a wide range of shear rates. The power law is given as:

\[
\tau = K(S^0)^n
\]  

dimensionless.

where, \( \tau \) is the shear stress (N/m²); \( K \) is the consistency coefficient (N · s⁻¹/m²); \( S \) is the shear rate (s⁻¹); \( n \) is flow behavior index (dimensionless). These constants can be used as an estimate of the polymer solution viscosity when there are no experimentally determined values.

The purposes of this work was to devise a power-law model for the flow behavior of polyacrylamide solution at various temperatures and concentrations, and to evaluate the effect of various factors on the flow pattern and finally to determine the applicability of this mathematical model.

**EXPERIMENTAL**

Polyacrylamide used in these experiments was obtained from EP grade of Wako Co. The materials and preparation of polyacrylamide solution experimental procedure were the same as those employed in the previous paper [7]. The rheological measurements were carried out using a Rotovisco RV 12 (Haake) concentric cylinder viscometer equipped with a M-500 type measurement attachment, which can transmit a maximum torque of 4.90 N-cm using NV-type pair coaxial cylinders. A thermostatic bath controls the working temperature within the range 20~50 ± 0.1 °C. Rotor speeds
were variable in the range 0-999 rpm, which enabled rheograms (shear stress versus shear rate diagram) to be constructed.

RESULTS AND DISCUSSION

Rheological behavior of polyacrylamide concentration

The rheological behavior of low concentration polyacrylamide solutions was studied in the 20-50ppm by weight of concentration range and in the temperature range of 20-50 °C. Figure 1 shows the experimental results of shear stress (τ) versus shear rate (SR), which indicate the non-Newtonian behavior of these solutions. Figure 1 shows the experimental results obtained for the polyacrylamide solution at 20 ppm at four different temperatures, the shear stress increased with increasing the shear rate. Similar levels were observed at other concentration also, for examples, 30, 40, 50ppm. It can be seen from the Figure 1 that at higher temperature, shear stress decreased and at higher concentration polyacrylamide solution shear stress increased. Hence, the polyacrylamide solution studied in this paper behaved as a non-Newtonian fluid. The values of the consistency coefficient and flow behavior index obtained using eqn.(1) by fitting the experimental results to non-Newtonian relationship. Figure 2 shows the experimental results obtained for the polyacrylamide solution at 30 °C at four different concentration, 20, 30, 40, 50ppm. The shear stress increased with increasing the shear rate. Similar levels were observed at other temperatures, 30, 40, 50 °C. It can be seen from the Figure 2 that at higher concentration polyacrylamide, shear stress increased and at higher temperature shear stress decreases.

Taking logarithms, eq.(1) becomes,

\[
\ln(\tau) = \ln(K) + n \ln(SR)
\]  

(2)
The ln(SR) dependence of ln(\(\tau\)) for the 30ppm at various temperature is given in Figure 3. From this dependence of the ln(K) and n, where evaluated by linear regression analysis of the points and are shown in Figure 3.

**Effect of temperature on the rheological characteristics**

The relationship between the temperature and rheological characteristics (i.e. consistency coefficient and flow behavior index) is expressed by the following Arrhenius relationship is shown in Figure 4.

\[
  n = n_0 e^{-E_a/RT} \quad (3)
\]

\[
  K = K_0 e^{E_a/RT} \quad (4)
\]

Where, \(n_0\) and \(K_0\) represent constants; \(E_a\) represents the activation energy (cal/g-mole), \(R\) represents the gas constant (cal/g-mole•K), and \(T\) is the absolute temperature, K. As shown in Figure 4, an increase in the temperature leads to decrease in a consistency coefficient and increase in flow behavior index.

The temperature dependence of the ln(n) and ln(K) for different polyacrylamide concentration solution is given in Figure 5. An Arrhenius-type dependence of ln(n) and ln(K) on temperature in the temperature range 20-50\(\degree\)C is observed. From this dependence, the activation energies were evaluated by linear regression analysis of the data points and are shown in Figure 5. It was found the energy is 0.245 and 3.61 kcal/g-mole, and independent of the concentration of polyacrylamide solution in the range of 20~50ppm.

**Effect of the concentration on the rheological characteristics**

The relationship between the polyacrylamide concentration and rheological characteristics (i.e. consistency coefficient and flow behavior index) is expressed by
different expressions, they are generally of the power-law type or the exponential-law type [7-9]:

\[
A = A_1 (C)^{a_1} \quad \text{where } A: n, K \quad (5)
\]

\[
B = B_2 \exp(a_2C) \quad \text{where } B: n, K \quad (6)
\]

In both the equations, \(n_i\) and \(K_i\) are flow behavior index and consistency coefficient, \(a_i\) and \(b_i\) are constants and \(C\) is the concentration in ppm. In order to calculate the different parameters of these equations, the \(\ln(C)\) dependence of the \(\ln(n)\) and \(\ln(K)\) for the different temperature by using eq.(5) is given in Figure 6. According to eq.(5), a straight line should result from \(\ln(n)\), \(\ln(K)\) and \(\ln(C)\). The dependence \(\ln(C)\) for different temperature is given in Figure 6 and illustrate that for those cases a straight line exists between the \(\ln(n)\), \(\ln(K)\) of the polyacrylamide solution and the \(\ln(C)\), where slopes are \(a_1\) & \(b_1\), respectively. From this dependence \(a_1\) & \(b_1\) were evaluated by linear regression analysis. These fits were obtained by least square method, showing in both the cases that the fit and the estimates of the parameters were significant at a probability level of 95%.

The concentration dependence of the \(\ln(n)\) and \(\ln(K)\) for the different temperature by using eqn. (6) is given in Figure 7. According to eqn. (6), a straight line should result from the \(\ln(n)\), \(\ln(K)\) and concentration. The dependence of concentration on different temperature is given in Figure 7 and illustrate that for those cases a straight line exists between the \(\ln(n)\) and \(\ln(K)\) of the polyacrylamide solution and the concentration where slopes are \(a_2\) & \(b_2\), respectively. From this dependence \(a_2\) & \(b_2\) were evaluated by linear regression analysis. These fits were obtained by least square method, showing in both the cases that the fit and the estimates of the parameters were significant at a probability level of 96%.

From the values of the regression coefficient obtained, the experimental results
of flow behavior and polyacrylamide concentration were fitted to the linear form of eqn. (3) & (4) by the least square method to obtain the estimates of the parameters of the model. The best fit was found when the exponential relation (eqn. (5) & (6)) was used for describing the effect on the rheological behavior index and consistency coefficient of the polyacrylamide solution.

### Combined effect of temperature and concentration on the rheological behavior

For practical engineering applications, it is useful to get a combined equation describing the combined effect of temperature and concentration on polyacrylamide solution rheological behavior.

From the results obtained in the earlier section, the following equations were proposed:

\[ \tau = K(SR)^n \]

\[ A = A_3 \exp(a_3C - Ea/RT) \] where A: n, K \hspace{1cm} (7)

\[ B = B_4 C^{a_4} \exp(-Ea/RT) \] where B: n, K \hspace{1cm} (8)

The rheological characteristics were fitted to these equations by multiple linear regressions calculated by Box [10] and computer program was used to calculate the linear regression. The values of the constants obtained are given in Table 1. The fit and the estimates of the constants are significant at a probability level of 98%. From the results obtained, it seems that eqn. (7) best describe the combined effect of temperature and concentration. Therefore, for the range of concentrations and temperatures used, a combined equation is proposed to describe the rheological characteristics of low concentration of polyacrylamide solution.

\[ \tau = K(SR)^n \] \hspace{1cm} (9)
Where, \( n=2.337 \exp(-0.00707C - 245/RT) \), \( K=2.446 \times 10^{-7} \exp(0.0639C + 3613/RT) \)
in which \( C \) represents the concentration, ppm; \( T \) represents the temperature in K.

A test was made on the applicability of eqn. (9). This was checked by plotting experimental \( \tau \) values versus calculated \( \tau \) values by using eqn. (9). This type of plot is shown in Figure 8, and it can be seen that for the most part remarkably straight line are formed. This results combined effect of temperature and concentration suggests that the eqn. (9) is reasonable to show the rheological behavior of low concentration polyacrylamide solution.

**CONCLUSION**

The effect of temperature and concentration on rheological behavior of polyacrylamide solution were examined in the temperature range 20~50 ℃ and concentration range 20~50ppm, using a coaxial cylinder viscometer. The rheological behavior of low concentration of polyacrylamide solution behaves as non-Newtonian fluids. It was found that the shear stress of low concentration of polyacrylamide solution decreased with increasing temperature and increased with increasing concentration. The consistency coefficient of polyacrylamide solution decreased and flow behavior index of polyacrylamide solution increased with increase in temperature. It was found that the effect of temperature on the consistency coefficient and flow behavior index of polyacrylamide solution of the different concentrations followed Arrhenius type relationship. Moreover, the consistency coefficient of polyacrylamide solution increased and flow behavior index of polyacrylamide solution decreased with increasing polyacrylamide concentration. The power law and exponential law were used to fit the experimental data. The rheological constants for the exponential law and power law models were determined. The empirical equation
\[ T = K(SR)^n \], where \( K = 2.446 \times 10^{-7} \exp(0.0639C + 3613/RT) \) and \( n = 2.337 \exp(-0.00707C - 245/RT) \), was given for describing the combined effect of temperature and concentration on the behavior properties of low concentration of polyacrylamide solution. It was found that the experimental data and the equation correlate well each other.

REFERENCES