Optimization of inverter placement for mass rapid transit systems using genetic algorithm

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Abstract—This paper has presented a methodology to solve the optimal planning of inverter substations for an electrified Mass Rapid Transit (MRT) system by using the genetic algorithm (GA). The mathematical models of power converters in traction substations have been derived for different operation modes. With the variation of power demand of train sets along the main line, the AC/DC load flow analysis has been performed to find the energy consumption and braking regeneration restoration at each power converting substation for system peak and off peak operation over the study period. The overall cost of power consumption, inverter investment, and service reliability are included in the objective function, and each feasible solution is expressed as a chromosome in the GA simulation process. The fitness is then enhanced by considering the diversity of chromosomes so that the global optimization during the solution process can be obtained. It is found that the energy regeneration, which has been resulted from braking operation of train sets approaching the next station, can be restored effectively by the optimal planning of inverters using the proposed genetic algorithm.

Index Terms—AC/DC Load Flow, Mass Rapid Transit (MRT), Genetic Algorithm(GA), Inverter Substation Planning, 12 Pulse Uncontrolled Rectifier

I. INTRODUCTION

The mass rapid transit (MRT) systems have been implemented in many metropolitans to solve the problems of public transportation and air pollution. Because the MRT serves the urban public transportation with short headway and heavy ridership, the distance between two stations is often less than 1km in downtown service area. Therefore the train set has to be operated with large torque to achieve quick acceleration during start up. When the train set approaches the next station, the hybrid braking mode is applied to achieve quick stop by applying the pneumatic mechanical friction braking and electric braking simultaneously. After the speed of train set has been reduced to a specified value, only the electric braking will be applied for full restoration of kinetic energy. It is expected that very high percentage of kinetic energy of train sets during braking stage can be converted to electricity energy by controlling the stator winding frequency of induction machines to be less than its rotor speed for induction generation. The regeneration power is then fed into the third rail to supply power for another train sets with propulsion operation. With the injection of such large amount of power regeneration in the power network, the voltage level of the third rail will become too high and power dissipater has to be applied to consume the surplus power. By this manner, the power regeneration is wasted to prevent the damage of carbine inverters due to abnormal voltage rise and the air conditioning burden is increased due to the temperature rise in the tunnel.

According to the energy consumption of MRT train sets for different operation modes [1], the regeneration kinetic energy is 53% of the total energy consumption will be released during the braking stage. By subtracting the loss of motion resistance and mechanic braking, there are 22% of the energy consumption which can be restored by electric regeneration. It is expected that very significant energy consumption can be recovered by the inverters installed at proper locations along the main line [2-4]. In this paper, the energy consumption of all traction substations is solved by executing the ac/dc load flow analysis with the growth of annual MRT ridership [5-6]. The yearly energy restoration of braking regeneration with different headways is calculated by using the mathematical model of power inverters.

To solve the problem of optimal inverter placement for MRT systems, the genetic algorithm (GA) is used in this paper to improve the efficiency of simulation process [7-10]. The overall cost of power consumption, inverter investment, and service reliability are included in the objective function, and all feasible solutions are expressed as chromosomes in the GA. The genetic operators which include crossover and mutation are then processed to produce individual of the population in a feasible space. The fitness is then enhanced by considering the diversity of population so that the global optimization during the solution process can be obtained.

II. PROBLEM DESCRIPTION AND MATHEMATICAL MODELS OF TRACTION SUBSTATION

In this paper, the energy consumption of MRT train sets for different operation modes are investigated and the braking energy restorations by inverters at traction substations are analyzed. The total energy consumption of an MRT system with the corresponding annual ridership by considering the stochastic operation characteristics of train sets are used to...
find the optimal placement of traction substations to enhance the operation efficiency of MRT systems. With the annual growth of passengers, the headways of train operation are reduced to comply with the service performance index. The power restoration of braking regeneration by inverters has to be calculated for the peak and off peak operation over the study period. To solve the power consumption of an MRT system and energy restoration by inverters, the mathematical modeling of traction substations and induction motors are included in the simulation of load flow analysis.

A. Schematic diagram of traction substations

The traction substation of an MRT system is consisted of parallel uncontrolled rectifiers with an inverter as shown in Fig. 1. The 12 pulses traction substation is formed by two sets of 6 pulse diode rectifier bridge with $30^\circ$ phase difference to provide the 750 Vdc power source to the third rail for train set operation. The inverter is applied to recover the braking regeneration power to the ac side of the traction substation. With the uncontrolled operation mode, the V-I characteristics of the rectifier is determined by its commutating impedance. On the other hand, the output characteristics of the inverter can be controlled by either maintaining the output voltage constant [11]. The mathematic models for different operation modes of the rectifier and transformer set have been derived by authors [6]. The voltage and current at the dc side of rectifiers are represented as function of commutation angles and postignition angles in equation (1), (2).

$$E_a = \frac{6 \sqrt{3} V}{\pi} \left( \frac{\sqrt{2 \pi}}{7} \sin \left( a_3 + \frac{\pi}{6} \right) - \sin \left( a_3 + \frac{\pi}{6} \right) \right) + \left( \frac{3 + \sqrt{3}}{4} \right) \sin \left( a_3 + \frac{\pi}{12} \right) - \sin \left( a_3 + \frac{\pi}{12} \right)$$

$$I_a = \frac{\sqrt{3} V}{X_c} + \frac{7}{6} \sin \left( a_3 + \frac{\pi}{6} \right) - \sin \left( a_3 + \frac{\pi}{6} \right)$$

The real power and reactive power at the ac side of the rectifier is derived as equation (3), (4).

$$P_r = \frac{12 \sqrt{3} V}{\pi X_c} \left[ \sin \left( \mu - \frac{\pi}{6} \right) \Phi_i (\mu) + \sin \left( \mu - \frac{\pi}{3} \right) \Phi_i (\mu) \right] + \sin \left( a_3 + \mu \right) \Phi_i (\mu) + \sin \left( a_3 + \mu - \frac{\pi}{3} \right) \Phi_i (\mu) - \sin \left( \mu - \frac{\pi}{6} \right) \Psi_i (\mu) - \sin \left( \mu - \frac{\pi}{3} \right) \Psi_i (\mu)$$

$$Q_r = \frac{12 \sqrt{3} V}{\pi X_c} \left[ \sin \left( \mu + \frac{\pi}{6} \right) \Psi_i (\mu) - \sin \left( \mu + \frac{\pi}{3} \right) \Psi_i (\mu) - \sin \left( a_3 + \mu \right) \Psi_i (\mu) + \sin \left( a_3 + \mu - \frac{\pi}{3} \right) \Psi_i (\mu) + \left( \mu - \frac{\pi}{6} \right) \left( 0.92405 \right) + \left( \frac{\pi}{3} - \mu \right) \left( 0.43745 \right) \right]$$

Here, $\mu$ is the interphase commutation angle and $\alpha_s$ is the postignition angle.

The voltage and current at the dc side of the inverter is expressed as equation (5), (6)

$$E_n = \frac{3 \sqrt{2} V}{2 \pi} \left[ \cos \gamma + \cos (\gamma + \mu) \right]$$

$$I_n = \frac{3 \sqrt{2} V}{2 \pi X_c} \left[ \cos \gamma - \cos (\gamma + \mu) \right]$$

The real power and reactive power at the ac side of inverter is solved as equation (7), (8).

$$P_n = \frac{9 N_i}{4 \pi X_c} \left[ 2 \sin \mu \sin (2 \gamma + \mu) \right]$$

$$Q_n = \frac{9 N_i}{4 \pi X_c} \left[ 2 \mu - 2 \sin \mu \cos (2 \gamma + \mu) \right]$$

, where $\gamma$ is the extinction angle.

B. Train control

In Fig. 1, a variable voltage and variable frequency (VVVF) inverter is installed in each train set to adjust the frequency and magnitude of ac voltage for the induction machines according to the speed code and passenger ridership. The variable magnitude and variable frequency voltage source is generated by inverting the 750 Vdc from the third rail. For Taipei MRT system, the operation modes of PWM, Quasi-Six steps and six steps are used to generate the trigger signal for the inverters to drive the train sets according to the speed range of 0-22 Km/hr, 22-42 Km/hr and 42-80 Km/hr respectively. The induction motors will be operated with modes of constant power and constant slip when the speed reaches more than 42 Km/hr in. When the train set approaches the next station, the constant power generation mode is used for the induction motors for speed above 56 Km/hr. By this way, the induction motors can be prevented from overloading and variable mechanical braking is applied to mix with the electric braking to provide sufficient braking effort. After the speed has been reduced below 56 Km/hr, the constant torque mode of inductor motors is applied to achieve full electric braking.

C. Propulsion power consumption of MRT train sets

To solve the power consumption of train sets along the main line by performing the train performance simulation, the time table and speed profile of train operation between two
substations is used to define the acceleration, coasting, deceleration and dwelling for each time snapshot of train operation. According to the ridership and traction characteristics of propulsion system, the power consumption and braking regeneration of train sets are determined by considering the motion equation of train sets in equation (9) [12]

\[ F_A = M_e a + R \]  

(9)

Where \( F_A \), \( M_e \), \( a \) and \( R \) represent the traction effort, train mass, desired acceleration and operation resistance respectively. Fig. 2 shows the power and speed profile of a train set operated between two stations. During start up, large amount of propulsion power is required for train acceleration. As speed reaches the specified value, the coasting mode is used to maintain the constant speed with small amount of motoring power. When the train set approaches the next station, the kinetic energy of train sets is recovered by electric braking with induction regeneration.

**D. Headway of train operation**

With the increase of ridership over the study years, the headways of train operation have to be adjusted to meet the requirement of service criterion. In this paper, the annual ridership for Tamshui line of Taipei MRT system, which has been solved by forecast of mass transportation, is used to determine the headways for peak and off peak operation in Fig. 3. Based on the ridership and proposed headway, the annual total power consumption and regeneration of whole MRT system over study years are solved.

**E. Objective function**

To determine the optimal capacity and installation locations of inverters for regeneration power restoration of an MRT system, the objective function is formulated by including the cost of inverter investment, benefit of energy restoration, and equivalent cost of service reliability.

1. **Energy consumption cost**

The ac/dc load flow analysis has been executed to solve the snapshot power consumption of each traction substations according to the power demand of all train sets. The cost of energy consumption by the MRT system is then determined by summing up the energy consumption of all traction substations. The total energy consumption cost \( EC \) of the MRT system for year \( k \) is expressed as equation (10).

\[ EC_{(hw)} = E_{(hw)} p \times NH_{p} \times W_{(hw)} p + E_{(hw)} op \times NH_{op} \times W_{(hw)} op + W_{dem} \times 12 \]  

(10)

where \( E_{(hw)} p \), \( NH_{p} \) and \( W_{(hw)} p \) represent the energy consumption, total peak hours and energy charge rate respectively for peak period. \( E_{(hw)} op \), \( NH_{op} \) and \( W_{(hw)} op \) are the energy consumption, total off peak hours and energy charge rate respectively for off peak period. \( W_{dem} \) is the monthly demand charge.

2. **Investment cost of filters**

The investment cost of inverters with auxiliary equipment such as protective relays and circuit breakers is also considered in the objective function. The time until and of redemption (TUER) is used to solve the annual equivalent cost of inverters; the operation/maintenance cost of an inverter is defined as fixed percentage of its procurement cost. For the \( j \)th inverter to be implemented, the investment cost \( C_{inv} \) is expressed in equation (11).

\[ C_{inv}(j) = I \times \frac{e}{100} \times j \]  

(11)

where \( I \) is the initial cost of inverter procurement and \( e \) is the redemption rate.

3. **Equivalent cost of service reliability**

The forced outage rate of traction substations is considered to derive the probability of traction station outage. When the total residual capacity of traction substations is less than the system peak power demand, the induced service reliability cost is described as function of capacity shortage in (12).

\[ W_{si} = \left( \frac{i}{n} \right) \lambda_{si} (1 - \lambda_{ai})^{i-1} \times L_{y} \times IC_{y} \]  

(12)

where \( i \) is the total traction substation units; \( n \) is the number of outaged traction substations; \( \lambda_{si} \) represent the probability of traction substation outage [13]. \( L_{y} \) and \( IC_{y} \) represent the system peak power demand shortage and cost of service loss respectively for year \( y \).

In this paper, the GA has been used to solve the optimal placement of inverters by considering the cost of investment and the benefit of energy restoration by braking regeneration. For the study period of \( ny \) years with \( nl \) candidate inverter stations, the total energy cost of an MRT system and the possible inverter investment cost are considered in the objective function in equation (13).

\[
\text{Minimize } OPT = \sum_{y=1}^{ny} (EC_{(hw)} + W_{si,y}) + \sum_{j=1}^{nl} C_{inv}(j)
\]

Subject to

\[ 1 \cdot V_{min} \leq V \leq V_{max} \]

\[ 2 \cdot V_{min} \leq V \leq V_{max} \]
Equation (14) and (15) are the voltage constraints of traction substations and train sets.

III. GENETIC ALGORITHM

The GA has been widely used to solve the optimization problems by applying the same operation principle of natural selection and population genetics. They operate on populations which consist of a number of individuals, each representing a particular selection of the values of the variables coded in binary form. GA begins by randomly creating its population. Each individual of the population represents a search point in the space of potential solution of given optimization problem, i.e. each individual represents a feasible solution to the problem. Feasible solutions are combined by a crossover operator to produce offspring, which expands the current population of solutions. Thus the individuals in the population are evaluated via the fitness function. The less fit individuals are eliminated to remain the same population size as the original one. Meanwhile a mutation operator is performed at a certain probability level to increase variation in search space. By favouring the mating of the more fit individuals, the more promising areas of the search space are explored. The process of crossover, evaluation, selection and mutation is repeated until a predetermined number of generations is reached or a satisfactory solution has been found.

In this paper, the GA is proposed to find the desired inverter planning so that the kinetic energy of train sets, which has been converted to electrically during braking stage can be restored properly. With the GA method, the objective function and feasible inverter planning strategies are represented as population in the GA. The genetic operators which include crossover and mutation are then processed to produce individual of the population in a feasible space. The fitness is then enhanced by considering the diversity of population so that the global optimization during the solution process can be obtained.

The data structure of genes can be depicted as shown in Fig 4, where TSS(i) represents the candidate traction substation, which may be considered for the installation of inverters. For an MRT power system with N possible strategies of inverter planning and with M traction substations, there will be N feasible solutions having M genes in the genetic pool.

![Fig. 4 Data structure of genes for optimal inverter placement process.](image)

The computation procedure of GA method is executed as follows:

Step 1: Create an initial population with randomly generated string and set up control parameters of genetic algorithm.

Step 2: Evaluate all of the individuals with the objection function.

Step 3: Select a new population from the old population based on the fitness of the individuals as given by the evaluation function.

Step 4: Apply genetic operations of mutation and crossover to members of the population for creation of new solution.

Step 5: Evaluate the newly created individuals.

Step 6: Check if termination criteria are met. If not, go to Step 3. If yes, output the results.

IV. NUMERICAL ANALYSIS AND GENETIC RESULTS

A. AC/DC load flow analysis for MRT systems

To illustrate the optimal inverter planning with GA method for the energy restoration of braking regeneration for MRT systems, the Tanshui line of Taipei Rapid Transit system has been selected for computer simulation. Fig. 5 shows the single line diagram of the MRT power system. There are 2 bulk power substations served by Taiwan Power Company (TPC) with double 161 KV circuits. Twelve traction substations (TSS) along the main line are used to convert 22 KVac to 750 Vdc by rectifiers and transformers to serve the propulsion power demand of train sets. With operation schedule of 6 minutes headway, there are 6 train sets to be operated on the up track and down track respectively. For the time snapshot in Fig. 5, 2 train sets are approaching the next stations with electric braking while the other 10 train sets are accelerating with motoring operation. Each traction substation is operated with there different modes according to the voltage level Vdc at the dc side of TSS substation. When Vdc less than the no load voltage 750 V, TSS is operated with rectification mode to provide the propulsion power to the third rail. When Vdc is between the range of 750 V and 850 V, TSS is turned off and there is no propulsion power output by the traction substation. When Vdc is above 850 V due to the injection of regeneration power by braking train sets, TSS is operated with inverting mode to convert the regeneration energy to the ac side of the substation.

![Fig. 5 Power system network of Taipei Tamshui Line at one particular snapshot.](image)

Fig. 6 shows the power demand profile of traction substation TSS3. It is found that the largest power consumption of 2817 kW occurs at the 72th snapshot. There are 8 snapshots with braking energy restoration by inverting operation with the maximum value of -3478 kW at the 86th snapshot. Such a significant amount of braking regeneration does provide good potential for inverter placement to enhance the operation efficiency of the MRT system.
Fig. 6 Power demand profile of traction substation TSS3.

Fig. 7 shows the voltage variation of the 3rd rail before and after the installation of inverter at TSS3. Without the inverter, the 3rd rail voltage will reach above 900Vdc due to the injection of large amount of power braking regeneration with the train sets approaching the next station. The power dissipation resistor has to be used to consume the surplus power regeneration. With the installation of inverter at the traction substation TSS3, the voltage level Vdc has been reduced to be less than 850V and power restoration has been obtained.

Fig. 7 Voltage variation of the 3rd rail before and after the installation of inverter at TSS3.

Fig. 8 shows the percentage of energy restoration by each traction substation. Fig. 8 shows the percentage of energy restoration by each traction substation with headway from 2 minutes to 12 minutes. The energy restoration is varied dramatically with the dynamic operation modes of train sets on the main line. Besides, effectiveness of braking energy restoration by inverters is different for different traction substation. For instance, TSS 12, which is located at the end of main line, has much better energy restoration as compared to TSS8 at the center of main line.

B. Optimal inverter placement

By applying the GA method, the optimal placement of inverters at the appropriate traction substations is derived to achieve the best cost benefit of inverter investment. In this paper, the investment cost and the energy restoration over the study years are considered in the GA algorithm. By executing train performance simulation with the annual ridership and the mathematical models of diode rectifiers and inverters, the voltage level and power consumption of traction substations along Tamshui line of Taipei MRT system are solved. The annual headways have been adjusted with the growth of passenger ridership as shown in Fig. 3. The investment cost of US$600,000 for an inverter with 3 MW capacity is used in equation (13) for the objective function. To achieve better performance of the GA algorithm during optimization process, the sizes of population is selected as 100 and the GA generation is designated as 100. The crossover rate and mutation rate are assumed to be 0.8 and 0.1 respectively. By performing the GA simulation to solve the inverter placement, the optimal locations for inverter installation are solved as traction substations TSS2, TSS3, TSS4 and TSS6 for Taipei of Tamshui line in Fig. 5. Fig. 9 shows the annual energy consumption of MRT train sets solved for the following study cases: power system with inverters at all traction substations and with inverters at the proposed locations respectively. It is found that the annual propulsion energy consumption of Tamshui line is increased with the growth of annual ridership and shorter headway over the study years. The installation of inverters will reduce the energy consumption by restoring the braking energy back to the ac power system to prevent the energy waste by power dissipation for over voltage control. Although only 4 inverters are proposed by the GA method, the energy restoration is very close to the system with 12 inverters.

Fig. 9 Annual energy consumption of Taipei Tamshui line.

Fig. 10 shows the annual cost benefit of optimal inverter placement by considering the investment cost and the resultant energy restoration. The cost benefit becomes negative if the yearly energy restoration obtained is less than the equivalent yearly cost of inverter investment. By installing inverters at all substations, the cost effectiveness is negative for years 2003 to 2005 and years 2014 to 2016, and the total cost benefit is US$0.3 M over the study period. On the other hand, the cost benefit is positive for each study year by the optimal placement of inverters at 4 traction substations proposed in this paper. It is found that the cost benefit obtained is US$0.12 M for year 1998 and the total cost benefit over the study period is US$1.78 M. Therefore the braking regeneration energy of MRT train sets can be restored effectively by installing the inverters at proper traction substations.
achieve the best cost benefit with the proposed genetic algorithm. It is found that significant braking regeneration energy has been restored by the properly installation of inverters to Taipei MRT system has been selected for computer simulation. The propulsion energy consumption, the investment cost of inverters, and the equivalent cost of service reliability are included in the objective function. The objective function and the feasible inverter placements are represented as chromosomes in the GA method. The fitness of individual in the population has been calculated and used for the selection of new individual in the genetic pool is the optimal solution which will result in the best cost benefit of inverter investment. To illustrate the effectiveness of the proposed GA method to solve the optimal traction substations for each study year. The propulsion traction substations for each study year. The propulsion

V. CONCLUSION

In this paper, the annual passenger ridership of Taipei MRT system has been considered to solve the dynamic power demand of train sets along the main line. The ac/dc load flow analysis has been executed to solve the energy consumption of traction substations for each study year. The propulsion energy consumption, the investment cost of inverters, and equivalent cost of service reliability are included in the objective function. The objective function and the feasible inverter placements are represented as chromosomes in the GA method. The fitness of individual in the population has been calculated and used for the selection of new individual in the population for crossover and mutation to enhance the diversity of population. The individual with the largest fitness in the genetic pool is the optimal solution which will result in the best cost benefit of inverter investment. To illustrate the effectiveness of the proposed GA method to solve the optimal traction substations for inverter placement, Tamshui line of Taipei MRT system has been selected for computer simulation. It is found that significant braking regeneration energy has been restored by the properly installation of inverters to achieve the best cost benefit with the proposed genetic algorithm.

VI. REFERENCES


