Design of Multi-Bandpass FIR Digital Filters by Genetic Algorithm Approach

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Abstract

This paper presents the design of multi-bandpass finite-duration impulse response (FIR) digital filters by the genetic algorithm (GA) approach. The unique advantage of this approach over the McClellan-Parks algorithm is that it is general enough to incorporate both time-domain and frequency-domain constraints. The simplicity and generality of this GA-design approach make it attractive for a wide variety of applications.

Keywords: multi-passband FIR digital filters, GA approach

I. Introduction

In the past twenty years, the most popular program for designing linear-phase FIR digital filters has been developed by McClellan and Parks [1]. Recently, the GA approach has been proposed to design FIR digital filters [2]. By minimizing a quadratic measure of the error, fitness, in the design bands, a chromosome of a population is evolved to get the filter coefficients after several generations. The unique advantage of this approach over the McClellan-Parks algorithm is that it is general enough to incorporate both time-domain and frequency-domain constraints. Furthermore, the GA approach has several practical advantages such as simplicity and easy implementation with satisfactory performance.

It is shown that the method is not only simple but also optimal in the least-squares sense. Comparing with the McClellan-Parks algorithm, both are optimal in the sense of different minimum norms of the error functions, but much better performance is obtained with the GA approach in most of the frequency bands except in the narrowband regions near the cutoff edges.

II. Genetic algorithms

The usual form of GA was described by Goldberg [3]. A chromosome is real-valued instead of binary bit strings in this paper.

II.1 Crossover operation

Crossover is the main genetic operator. It operates on two chromosomes at a time and generates offspring by combining both chromosomes’ features. For direction-based operators, problem-specific knowledge is introduced into genetic operation in order to produce improved offspring.

Direction-based crossover uses the values of objective function in determining the direction of genetic search [4]. The operator generates a single offspring \( B' \) from two parents \( B_1 \) and \( B_2 \) according to the following rule:

\[
B' = r \cdot (B_1 - B_2) + B_i
\]

where \( r \) is a random number between 0 and 1. It also assumes that the parent \( B_i \) is not worse than \( B_j \); that is, \( \text{fitness}(B_i) \geq \text{fitness}(B_j) \) for maximization problems and \( \text{fitness}(B_i) \leq \text{fitness}(B_j) \) for minimization problems. In this paper, the latter case, minimization, is used to deal with the multi-bandpass FIR digital filters design.

II.2 Mutation operation

Mutation is a background operator which produce spontaneous random changes in various chromosomes. A simple way to achieve mutation would be to randomly generate chromosomes. In this paper, the mutation rate is set to be 1%.

II.3 Selection operation
The principle behind GAs is essentially Darwinian natural selection. When selection performs on enlarged sampling space, both parents and offspring have the same chance of competing for survival. An evident advantage of this approach is that we can improve GA performance by increasing the crossover and mutation rates. We do not worry that the high rate will introduce too much random perturbation if selection is performed on enlarged sampling space. According to the fitness values, a new generation is formed by selecting the better chromosomes from the parents and offspring, and rejecting others so as to keep the population size constant. After several generations, the algorithm converges to the best chromosome, which represents the optimal solution to the problem.

The population size plays the crucial role of representing the solutions to the problem at hand. If it is too small, many chromosomes that would have been useful are never tried out. On the other hand, it will take a lot of computation time if it is too large. In this paper, the proper population size is set to be 100 through the repeated experience.

### III. Preview

A typical FIR digital filter can be characterized by the transfer function
\[
H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}
\] (2)
where \(N\) is the filter length of the impulse response, \(h(n)\), and its frequency response is represented by
\[
H(e^{j\omega}) = \hat{H}(\omega)e^{jL\omega2\pi(N-1)\omega/2}
\] (3)
where \(\hat{H}(\omega)\) is the magnitude response which is a real-valued function and \(L\) is equal to 0 or 1. According to the symmetric properties of the impulse response and the filter length, there are four cases to be taken into account [5].

Case 1: Symmetric impulse response and odd length:
\[
L = 0 \quad \text{and} \quad \hat{H}(\omega) = \sum_{k=1}^{N/2} b(k)\cos(k\omega).
\] (4)
where \(a(0) = h((N - 1)/2)\) and \(a(k) = 2h((N - 1)/2 - k)\) for \(k = 1, 2, \ldots, (N - 1)/2\).

Case 2: Symmetric impulse response and even length:
\[
L = 0 \quad \text{and} \quad \hat{H}(\omega) = \sum_{k=1}^{N/2} b(k)\cos((k - 1/2)\omega).
\] (5)
where \(a(k) = 2h(N/2 - k)\) for \(k = 1, 2, \ldots, (N - 1)/2\).

Case 3: Anti-symmetric impulse response and odd length:
\[
L = 1 \quad \text{and} \quad \hat{H}(\omega) = \sum_{k=1}^{(N-1)/2} b(k)\sin(k\omega).
\] (6)
where \(a(k) = 2h((N - 1)/2 - k)\) for \(k = 1, 2, \ldots, (N - 1)/2\).

Case 4: Anti-symmetric impulse response and even length:
\[
L = 1 \quad \text{and} \quad \hat{H}(\omega) = \sum_{k=1}^{(N-1)/2} b(k)\sin((k - 1/2)\omega).
\] (7)
where \(a(k) = 2h(N/2 - k)\) for \(k = 1, 2, \ldots, (N - 1)/2\).

The least-squares approach to these filter designs is to formulate an objective error function, fitness, as below
\[
E = \int_{\omega}^{\omega} |D(\omega) - \hat{H}(\omega)|^2 \, d\omega
\] (8)
where \(D(\omega)\) is the desired magnitude response and \(R\) represents the region of design bands.

### IV. Design of multi-bandpass filters

Multi-bandpass filters can be only designed by using symmetric impulse response sequence, i.e., Cases 1 and 2 designs are suitable to use.

Define
\[
B = \begin{bmatrix} b(0) & b(1) & \cdots & b((N - 1)/2) \end{bmatrix}^T, \quad \text{Case 1},
\] (9)
and
\[
C(\omega) = \begin{bmatrix} 1 & \cos(\omega) & \cdots & \cos((N - 1)\omega/2) \end{bmatrix}^T, \quad \text{Case 1},
\] (10)
where superscript $T$ stands for the vector transpose operation. Then $\hat{H}(\omega)$ can be rewritten as

$$\hat{H}(\omega) = C^T(\omega)B\hat{.}$$

Suppose the multi-bandpass filter has $I$ multiple passbands and $J$ multiple stopbands. The error function, fitness, with $k$-th chromosome of the $j$-th stopband is

$$E_{sj} = \int_{R_j} |\hat{H}_j(\omega)|^2 d\omega$$

$$= \int_{R_j} |C^T(\omega)B_j|^2 d\omega$$

(12)

where $R_j$ represents the interval of the $j$-th stopband. Similarly, the error function, fitness, with $k$-th chromosome of the $i$-th passband is

$$E_{pi} = \int_{R_i} |D(\omega) - \hat{H}_i(\omega)|^2 d\omega$$

$$= \int_{R_i} |D(\omega) - C^T(\omega)B_i|^2 d\omega$$

(13)

where $R_i$ represents the interval of the $i$-th passband. So the total error with $k$-th chromosome

$$E_k = \alpha_j E_{sj} + \alpha_j E_{sj} + \cdots + \alpha_j E_{sj}$$

$$+ \beta_i E_{pi} + \beta_i E_{pi} + \cdots + \beta_i E_{pi}$$

(14)

where $\alpha_j$ and $\beta_i$ are the weighting constants of the $j$-th stopband and the $i$-th passband, respectively. In general, the larger the parameter is, the better the performance of its corresponding band is. $\alpha_j$ and $\beta_i$ can be also a function $\omega$. After applying the proposed GA approach, the optimal chromosome $B$ corresponding to the smallest fitness value can be obtained.

Example: In this example, we deal with the design of length $N = 32$ bandpass filter with passband $[0.26\pi, 0.46\pi]$ and stopbands $[0, 0.16\pi]$, $[0.6\pi, \pi]$. By using the GA method with 5,000 generations, the best chromosome corresponding to the smallest fitness value at each generation is shown in Fig. 1. The optimal solution is obtained in the 2,096th generation. This method is not only simple but also effective. It takes about 41.13 seconds of CPU time to deal with this problem based on an Intel Celeron 1.2GHz system with MATLAB 4.2c software package. When $\alpha_i = 3$, $\beta_i = 1$, and $\alpha_j = 5$ are used, the magnitude response and error are shown in Fig. 2(a) and (b), respectively, in which the dotted lines represent the responses of that designed by McClellan-Parks program. Notice that except the bandedges, the performance of the GA approach is much better.

8. Conclusions

This paper presents the GA approach for designing linear-phase FIR digital filters. Both time-domain and frequency-domain constraints are so easy to incorporate in the design procedures that a large class of linear-phase filters can be designed efficiently. Multi-bandpass filters are presented to demonstrate the simplicity and generality of this approach.

References

Fig. 1 Fitness

Fig. 2 Design of a band-pass filter (a) magnitude response (b) magnitude error.
(solid line: GA approach, dotted line: McClellan-Parks algorithm)

Fig. 2(a)