INTRODUCTION

Groundwater is an important water resource in Taiwan. Owing to the industrial development of Taiwan in the recent years and in dealing with the demand to increase the standard of living, the application degree of groundwater relatively increases and the demand for groundwater resources development is also growing with each passing day. Besides, there are also some natural calamities. To reasonably manage groundwater extracted amount, accurate control of the groundwater recharge is a very important basis.

Groundwater recharge is an important variable in regional-scale hydrologic models and aquifer-system analysis [12]. With the purpose of assessing recharge, most cases of recharge studies were estimated by multiplying the magnitude of water-level fluctuations in wells, to the specific yield of the aquifer material [3, 14]. Processes of infiltrating recharge and fluxes have been discussed based on the water balance concept (Avon and Durban 1994; Arnold and Allen 1999; Arnold et al. 2000; Ketchum et al. 2000; Louie et al. 2000; Otto 2001; Yeh et al. 2004). Currently, standard techniques of estimating regional recharge most often involve (1) applying a soil moisture budget, where the moisture content of the soil is tracked through time [8, 11, 12, 29–32, 35] or (2) estimating the based-flow under the stream hydrograph and that groundwater evaporation is negligible [5, 10, 13, 21–23, 28].

The application of the first approach is proposed to estimate the infiltration, runoff, evapotranspiration, and groundwater recharge in the watershed, where the moisture content of the soil is tracked through time. Secondly, the groundwater recharge was also estimated by the model of the base-flow-record estimation, with the assumption that groundwater evaporation is negligible. In addition, since the analyzed base-flow trends are high, when executing model analysis, the depths of infiltration estimated by stable-base-flow analysis is used to obtain more reasonable groundwater recharge value. The coefficients of groundwater recharge by the precipitation in the Ching–Shui watershed estimated from the established soil moisture budget model and the base-flow model were 12.40% and 9.92%, respectively. Comparison show the result of both models to be close.

Abstract—The main purpose of this paper is to apply a water balance concept with two models in the Ching–Shui watershed to describe the groundwater recharge. First of all, a soil moisture budget model is established to estimate the infiltration, runoff, evapotranspiration, and groundwater recharge in the watershed, where the moisture content of the soil is tracked through time. Secondly, the groundwater recharge was also estimated by the model of the base-flow-record estimation, with the assumption that groundwater evaporation is negligible. In addition, since the analyzed base-flow trends are high, when executing model analysis, the depths of infiltration estimated by stable-base-flow analysis is used to obtain more reasonable groundwater recharge value. The coefficients of groundwater recharge by the precipitation in the Ching–Shui watershed estimated from the established soil moisture budget model and the base-flow model were 12.40% and 9.92%, respectively. Comparison show the result of both models to be close.

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flow and base-flow is assumed to be two major constituents of stream flow.

The objective of this paper is to adapt the above-mentioned two water budget models to estimate the recharge at the Ching–Shui watershed in Taiwan. Soil water budget model was first established to estimate the infiltration, runoff, transpiration, and the groundwater recharge in the Ching–Shui watershed. The soil water parameters were derived from laboratory and in situ experimental tests, and hydrological data were collected. The established water budget model was combined with the base-flow estimation model to analyze the hydrogeological conductions in the Ching–Shui watershed.

**METHODOLOGY**

**Soil Moisture Budget Model**

According to the schematic illustration (Fig. 1) of soil moisture budget for a profile, the total precipitation in a rainfall event can be expressed as

\[ P_{\text{cum}}(t) = i_{\text{cum}}(t) + q_{\text{cum}}(t), \]

where \( P_{\text{cum}}(t) \) is the cumulative precipitation, \( i_{\text{cum}}(t) \) is the cumulative infiltration, and \( q_{\text{cum}}(t) \) is the cumulative infiltration excess runoff over the time period \( t \). The relation between infiltration, evapotranspiration, recharge, saturation excess runoff (representing runoff resulting from the super-saturation of the soil profile), and storage change can be written as

\[ i_{\text{cum}}(t) - q_{\text{cum}}(t) = E_{\text{cum}}(t) + q_{\text{cum}}(t) + \Delta \phi, \]

where \( q_{\text{cum}}(t) \) is the cumulative saturation excess runoff, \( E_{\text{cum}}(t) \) is the cumulative evapotranspiration, \( q_{\text{cum}}(t) \) is the cumulative recharge, and \( \Delta \phi \) is the change in storage.

To derive mathematical models for the components in equations (1) and (2), the following assumptions are imposed: (1) the soil profile has uniform hydraulic properties; (2) the infiltrated water is instantaneously and uniformly distributed (i.e., well-mixed approach); (3) a linear relation exists between evapotranspiration and the effective saturation of the soil profile.

For the surface infiltration rate, Philip [26] provided a surface infiltration function:

\[ i(t) = \frac{1}{2} S \phi(0) t^{-1/2} + a k_s, \]

where \( S \) is the sorptivity, \( \phi(0) (0 \leq \phi \leq 1) \) is the initial effective saturation degree, \( t \) is the time, \( a \) is a constant, and \( k_s \) is the saturated hydraulic conductivity. The parameter \( \phi \) is the effective saturation degree, which is defined as the follows:

\[ \phi = (\theta - \theta_s)/(\theta_s - \theta_r), \]

where \( \theta \) is the volumetric moisture content, \( \theta_s \) is the saturated volumetric moisture content and \( \theta_r \) is the residual volumetric moisture content.

Using the Brooks and Corey [6] relationships, the soil water retention curve and hydraulic conductivity function can be described as follows for the soil water retention curve:

\[ \phi(\psi) = \left(\frac{\psi}{\psi_s}\right)^{\lambda}, \quad \psi < \psi_s, \]

\[ \phi(\psi) = 1, \quad \psi_s \leq \psi < 0, \]

for the hydraulic conductivity function:

\[ k(\phi) = k_s \phi^{(2 + 3\lambda)/\lambda}, \]

where \( \psi_s \) and \( \lambda \) are fitting parameters. The parameter \( \psi_s \) is the air entry value, \( \lambda \) is referred to as the pore size distribution index, and \( k \) is the unsaturated hydraulic conductivity.

Mein and Larsson [20] outlined the time compression approximation (TCA) method. It was also discussed in detail by Parlange et al. [25]. This method combined with Philip’s solution is more reliable for simulating rainfall infiltration for a short time period. When the infiltration rate, \( i \), is equal to rainfall intensity, \( P \), the equivalent time \( t_e \) can be derived from (3). In fact, the ground surface ponding does not occur at this time. The actual time to pond, \( t_p \), is longer than \( t_e \). An expression for \( t_p \) is obtained by equating the cumulative infiltration resulted from the flux-controlled infiltration up
to \( t_c \) to the cumulative infiltration that would result from head-controlled infiltration up to \( t_c \):

\[
\int_0^{t_p} P dt = \int_0^{t_c} i(t) dt.
\]  

(7)

The ponding time \( t_p \) decreases with increasing rate \( P \) and/or decreasing saturated conductivity \( k_s \). According to equation (3), after ponding, infiltration continues to occur. This situation happens when \( t > t_c \). In addition, it necessitates a shift of the temporal axis by \( t = (t_p - t_c) \). The subcript \( c \) is used to express two somewhat confusing yet commonly used terms—“compression” and “condensation.” The formula for the infiltration rate becomes:

\[
i(t) = \begin{cases} 
\frac{P}{2} & t \leq t_p, \\
\frac{1}{2} S(\phi) [t - t_c]^{-1/2} + a_k & t > t_p,
\end{cases}
\]

(8)

Cumulative infiltration, \( i_{\text{cum}} \), up to time, \( t \), can be derived by integrating equation (8) when \( t_c = t_p - t_c \). It yields:

\[
i_{\text{cum}}(t) = \begin{cases} 
P t & t \leq t_p, \\
P t_p + Ak_s^1/2 (t - t_c)^{1/2} + a_k (t - t_p) & t > t_p,
\end{cases}
\]

(9)

where \( A \) is a parameter related to sorptivity. Then, considering the soil profile depth, \( d_s \), in which a uniform moisture profile exists with the assumption of instantaneous redistribution, at any time \( t \) during rainfall, the state of soil moisture in the soil water reservoir is given by:

\[
\phi(t) = \phi_0 + \frac{i_{\text{cum}}(t)}{d_s (\theta_s - \theta_r)}.
\]

(10)

The total amount of infiltration excess runoff, \( q_{\text{ie}} \), up to time \( t \) is given by:

\[
q_{\text{ie}}(t) = P t - i_{\text{cum}}(t).
\]

(11)

The reservoir may super-saturate (\( \phi(t) \geq 1 \)) in model simulation. When this occurs, the amount of saturation excess runoff, \( q_{\text{se}} \), up to time \( t \) equals:

\[
q_{\text{se}}(t) = d_s (\theta_s - \theta_r) [\phi(t) - 1].
\]

(12)

Theoretically, \( q_{\text{se}} \) is attained when the soil profile depth is very shallow and/or the rainfall is unceasing. This seldom occurs in actual cases. The evapotranspiration and groundwater recharge estimation in the unsaturated zone results in that effective saturation degree can be expressed as:

\[
\phi(t) = \left[ \frac{\phi_0}{\phi_0 + \frac{k_s}{k_s}} \right] \exp \left[ \frac{c E_p}{d_s (\theta_s - \theta_r)} \right] - \left[ \frac{k_s}{E_p} \right]^{-1/c},
\]

(13)

where \( c = (2 + 2\lambda) / \lambda \). The cumulative amount of evapotranspiration up to time \( t \) can be expressed as:

\[
E_{\text{cum}}(t) = d_s (\theta_s - \theta_r) \phi_0 \epsilon_0 \frac{1}{\phi_0} \left[ 1 - \frac{\phi_1(t)}{\phi_0} \right]
\]

\[
+ \sum_{n=1}^{\infty} \frac{(1 - \frac{\phi_r(t)/\phi_0}{\phi_0})^{nc+1}(1 - \epsilon_0^n)}{n(n^c + 1)B(c^{-1}, n)}
\]

(14)

where \( B(x, y) \) is beta function, \( B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x + y)} \), and \( \Gamma(x) \) is gamma function. \( \Gamma(v) = \int_0^\infty x^{v-1} \exp(-x) dx \), \( \phi_1(t) \) can be derived from equation (11) when \( k_s = 0 \).

The accumulation of groundwater recharge can be readily obtained from mass conservation condition:

\[
q_{\text{g}}(t) = d_s (\theta_s - \theta_r) [\phi_0 - \phi(t)] - E_{\text{cum}}(t).
\]

(15)

This model can be briefly summarized as follows: for soil wetting, the cumulative infiltration can be derived from equation (9), and the cumulative runoff can be derived from equations (11) and (12); for drainage in the unsaturated zone, the cumulative evapotranspiration and groundwater recharge can be derived from equations (14) and (15).

**Base-Flow Estimation**

**Base-flow separation.** Various techniques have been used to estimate groundwater discharge from the streamflow hydrograph. Most of these techniques involve a considerable degree of subjectivity in their applications [9]. Horton [13] described a method of shifting a horizontally “normal depletion curve” across a hydrograph. It is noted that segments of the hydrograph which coincide with this curve represent periods of streamflow equal to groundwater discharge. Thus, estimating groundwater discharge during periods of surface runoff can be done by simply connecting the points where hydrograph departs from the normal depletion curve. Barnes [5] separated surface flow, storm seepage, and base flow by assigning a distinct “depletion factor” to each part. Kulandaiswamy and Seetharaman [16] pointed out that Barnes’ method may not be reliable to separate streamflow into three parts and they separated the hydrograph into direct runoff and base flow. In some applications, investigators have used characteristic curves of groundwater discharge in conjunction with records of precipitation, snowfall, temperature, and groundwater levels [23].
Flow as completely groundwater discharge (while surface runoff is negligible) can be based on the antecedent recession. Linsley et al. ([18]) proposed the empirical relation

\[ N = A_i^{0.2}. \]  

(16)

This relation gives the time base of surface runoff \( (N/d) \) as a function of the drainage area \( (A_i) \) upstream from a stream-flow-gaging station, in square miles. The time base of surface runoff is the number of days after a peak in the hydrograph of streamflow while the component of flow attributed to surface runoff (including the bulk of interflow) is considered negligible. A part of the streamflow hydrograph may thus be considered to be due completely to groundwater discharge, if it is preceded by a period of recession equal to or greater than \( N \).

The base-flow-record estimation explained here is a form of streamflow separation. Rutledge [27] developed this method first based on the antecedent streamflow recession. The principles of this method are as follows: (1) daily data on streamflow are required; (2) linear interpolation is used to estimate groundwater discharge during the period of surface runoff.

The scheme shows a flow diagram of the steps analyzed by the method of base-flow estimation. The requirement of the antecedent recession is met for the day in question if, for the part of the daily mean streamflow record that includes all days that precede the day in question by \( N \) days or less, the streamflow on each of these days is greater than or equal to the streamflow on the day that follows, where \( N \) is the time base of surface runoff.

Steps of the method are as follows (see scheme). First, a one-dimensional array of the daily mean streamflow data is filled. This array is searched for days that fit the requirement of the antecedent recession. On each of these days, groundwater discharge is taken equal to streamflow, as long as it is not followed by a daily decline of more than 0.1 log cycle. According to Barnes (1939), a daily decline more than 0.1 log cycle could indicate interflow (stormflow) or surface flow. The array is searched again, and it is determined by linear interpolation of the groundwater discharge in the remaining days. For some streamflow records, this interpolation can cause the calculated groundwater discharge to exceed streamflow for a few days on the record. The last step of the procedure is to correct this error.

Analysis of steady-state base flow. In the recharge and discharge behavior of a groundwater system, base flow may be subjected to the influence of rainfall of the year in short-term aspect, but in long-term aspect, the discharge flow of groundwater system is a stable value. Hence, this paper once again uses base-flow separation result to carry out the separation of groundwater recharge to estimate steady-state base flow [17].

To obtain the steady-state base flow, the monthly base-flow in the analyzed period, which is separated from streamflow, is used in the analysis of steady-state base flow. The analysis processes are as follows: (1) Obtain monthly base flow through base flow separation. (2) Total and average the monthly base-flow of each year to obtain long-term monthly average base-flow. (3) Sort the long-term monthly average base-flow by quantity (from the biggest to the smallest) and accumulate it to obtain the trend of monthly accumulated base-flow and its accumulated base-flow. (4) Use the trendline of accumulated base-flow to determine the rising point of base-flow and obtain the stable-base-flow days. (5) Execute linear regression of the accumulated base-flow within stable-base-flow days and use the extrapolation of linear regression equation to obtain annual steady-state base flow, which is annual depths of infiltration.

STUDY AREA

Two models of water balance described in previous section were applied to estimate the groundwater recharge on Ching–Shui watershed in Taiwan. The area of Ching–Shui watershed is about 421.4 km², and the length of the river is about 47 km. It is situated within latitudes 23°34′–23°46′N and longitudes 120°36′–120°49′E. As shown in Fig. 2, this area is located within the central region of Taiwan. The annual rainfall in this area is around 2100 mm, and 78% occurs from April to October [7].

ANALYSIS RESULTS

Soil Moisture Budget Analysis

In accordance with the Map of Taiwan Groundwater Resources and Modulus of Groundwater Flow by the Water Resource Agency (2003), the Ching–Shui watershed, located in central Taiwan, was divided into four areas shown in Fig. 2. Several experimental tests were carried out to obtain the soil water parameters shown in Table 1. The collected hydrological data were the precipitation, duration of rainfall, meteorological conditions, antecedent moisture, and soil hydraulic properties. Then, the soil moisture budget model as discussed previously was applied to estimate the infiltration, runoff, evapotranspiration, and the groundwater recharge. Figures 3a–3d show the comparison of the average annual infiltration, runoff, evapotranspiration, and recharge from 1992 to 2001 in areas (1), (2), (3) and (4), respectively. Table 2 shows the results in percent ratio in terms of the rainfall for each area. It indicates that the
ratio of recharge to rainfall in Table 2 was positively related to hydraulic conductivity in Table 1.

**Base Flow Analysis**

The method of the base-flow estimation aims at separating the base flow from a streamflow hydrograph in order to evaluate the discharge drained from groundwater to a stream. The method is based on the assumption that base flow (groundwater discharge) is equal to streamflow in days that meet the requirement of antecedent recession. For the days that do not meet this requirement, this method calculates the base flow by linear interpolation, and separates the streamflow hydrograph into two parts: the base flow and surface runoff. The duration of the recession period is calculated from equation (16).

Detailed steps of the method of the base flow estimation are presented in Fig. 1. To estimate the annual groundwater recharge rate, we collected daily streamflow and rainfall of the Toong–Tour gauging station in the Ching–Shui watershed for the time period from 1981 to 2000. The results of base flow estimation are shown in Table 2. The number represents the ratio of recharge to rainfall.

![Fig. 2. The Ching–Shui watershed divided into areas 1–4. The number represents the ratio of recharge to rainfall.](image-url)
shown in Fig. 4 indicate the recharge depths range in 65–180 cm/year. Figure 5 shows the result of streamflow separation at Toong–Tour gauging station, where groundwater discharge is equal to streamflow when curves coincide. Figure 6 shows monthly average streamflow and monthly average base flow at Toong–Tour gauging station of Ching–Shui watershed for 1981–2000.

Lee et al. [17] pointed out that the groundwater recharge might be overestimated when base flow separation is applied in a mountainous region, such as Taiwan. In order to overcome this, the steady-state base flow analysis was applied to the base flow data. The results show that such analysis can yield more reasonable estimates. In these cases, the minimum accumulation of recharge depths in the first four months in each year was fitted by linear regression since it is a better representative of the steady-state period of base flow. Figure 7 indicates that the annual recharge depth was 20.82 cm/year, and the groundwater recharges divided

Table 1. Input mean soil water parameters derived from laboratory experimental results for four areas

<table>
<thead>
<tr>
<th>Area</th>
<th>$K_s$, mm/d</th>
<th>$\theta_s$</th>
<th>$\psi_s$, mm</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>46.50</td>
<td>0.413</td>
<td>571.2</td>
<td>0.423</td>
</tr>
<tr>
<td>2</td>
<td>30.83</td>
<td>0.430</td>
<td>504.7</td>
<td>0.360</td>
</tr>
<tr>
<td>3</td>
<td>20.31</td>
<td>0.428</td>
<td>376.0</td>
<td>0.350</td>
</tr>
<tr>
<td>4</td>
<td>60.29</td>
<td>0.423</td>
<td>327.1</td>
<td>0.364</td>
</tr>
<tr>
<td>Average</td>
<td>39.48</td>
<td>0.424</td>
<td>444.8</td>
<td>0.374</td>
</tr>
</tbody>
</table>

Table 2. The percent ratio of annual infiltration, runoff, evapotranspiration, and recharge related to rainfall in four areas from 1992 to 2001

<table>
<thead>
<tr>
<th>Area</th>
<th>Infiltration</th>
<th>Runoff</th>
<th>Evapotranspiration</th>
<th>Recharge</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>53.8</td>
<td>46.2</td>
<td>38.4</td>
<td>16.8</td>
</tr>
<tr>
<td>2</td>
<td>45.3</td>
<td>54.7</td>
<td>37.6</td>
<td>8.7</td>
</tr>
<tr>
<td>3</td>
<td>34.9</td>
<td>65.1</td>
<td>32.7</td>
<td>3.2</td>
</tr>
<tr>
<td>4</td>
<td>58.1</td>
<td>41.9</td>
<td>38.3</td>
<td>20.9</td>
</tr>
<tr>
<td>Average</td>
<td>48.0</td>
<td>52.0</td>
<td>36.8</td>
<td>12.4</td>
</tr>
</tbody>
</table>

Fig. 3. The average of (1) annual precipitation, (2) infiltration, (3) runoff, (4) evapotranspiration, and (5) recharge from 1992 to 2001 (a–d) in areas 1–4, respectively.
by precipitation was 9.92% in the Ching–Shui watershed.

**CONCLUSIONS**

This paper presents a conceptual model of water balance in a watershed to estimate groundwater recharge in the case of the Ching–Shui watershed. A soil moisture budget model was developed to estimate groundwater recharge, infiltration, runoff and evapotranspiration in the unsaturated zone using soil hydraulic properties and meteorological data. The model offers an efficient and rapid procedure for estimating these hydrologic components under a variety of climate conditions and soil hydraulic properties for long-term simulation. In addition, this paper also uses base flow estimation of streamflow information to evaluate groundwater recharge rate for the Ching–Shui watershed. From the result of base flow separation, we cannot identify the days with steady-state baseflow in each watershed; hence, this leads to overestimation of groundwater recharge. However, the use of steady-state base flow analysis to choose the minimum accumulation of depths of infiltration in the first four months for the construction of linear regression, since these months are best representatives of the steady-state streamflow, yields better results. The coefficients of groundwater recharge by precipitation in the Ching–

![Fig. 4. Base flow separation from streamflow hydrograph in the Toong–Tour gauging station in (a) 1981–1990 and (b) 1991–2000.](image-url)
Fig. 5. Result of streamflow separation at Toong–Tour gauging station. Temporal base flow is 3 day. (1) Mean daily channel flow; (2) estimated mean daily base flow.

Fig. 6. Result of mean monthly base flow separation at Toong–Tour gauging station of Ching–Shui watershed, 1981–2000. (1) Channel flow; (2) base flow.

Fig. 7. Result of steady-state base flow analysis.
Construct three parallel 1-dimensional arrays:
1. Streamflow
2. Base flow
3. “ALLGW”

Read a data file of daily mean streamflow and assign values to Streamflow.
Assign all values of ALLGW = 0.

Locate all days that fit the antecedent recession requirement (see EXPLANATION).
On these days, reassign ALLGW=* and assign Base flow = Streamflow.

Locate each day when ALLGW=*. If it is followed by a daily decline of the log of streamflow exceeding 0.1, then reset ALLGW = 0.

Locate all days when ALLGW = 0. For these days, calculate the log of the base flow by linear interpolation between (1) the log of base flow of the closest preceding day when ALLGW=*, and (2) the log of base flow of the closest following day when ALLGW=*. Assign a value to base flow accordingly.

Are their any days when the base flow exceeds the streamflow?

If yes, locate all intervals where the value of ALLGW is continuously equal to 0 and where there is at least one day when base flow > streamflow. Find the day that exhibits the largest “log of base flow minus log of streamflow,” and reassign ALLGW = * for this day.

STOP

Fig. 8. Flow diagram showing the procedure of streamflow separation. Base flow is considered to be the groundwater discharge.

Shui watershed estimated from established soil moisture budget model and base flow model were 12.40% and 9.92%, respectively. The result of both models is quite close with the difference as small as 2.48%.

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REFERENCES


