A Computerized Approach to the Design of Automobile Suspension System

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Abstract

The design of automobile suspension system has been rebellious and crucial to the comfortableness and manipulation of the vehicle. This study not only proposes the dynamic model of a moving automobile mathematically, but also prepares the MATLAB program to simulate and analyze the dynamic system comprehensively. Additionally, the average data from several popular domestic small sedans are simulated with variant ground situations. Through the comparison and discussion of the simulated results, the compensation problem in between the comfortableness and manipulation with respect to different ground situations becomes observable and solvable. This study surely provides the applicable criteria to the design and development in this field, as well as contributes the valuable tool in analyzing and simulating the automobile suspension system.

Keywords: suspension system; spring; damper; spring stiffness; damping coefficient
1. Introduction

The main purpose of the automobile suspension system is to decrease the vibrations which cause from rugged road conditions. It plays a key role in the comfortableness and manipulation with respect to different ground situations. And people always hope to possess a nice vehicle with an excellent performance of suspension system [2]. The automobile suspension system is compose of springs, frame, wheels, cushions etc. [3] The basic requirements of a vehicle are with a lower natural vibration frequency and limited vibration amplitude that the passenger can sustain [4]. Elasticity of an automobile suspension system can make body and wheels of car with a lower vibration amplitude and fast vibration decay, and keep the body of car steadier [5]. The spring is the stronger in stiffness the more can support heavier weight. But if will make the car be suitable for every weight, should use the spring with several sections [6]. Stretch and shrinkage of an automobile suspension system just absorb and store energy, so can’t totally absorb the vibration wave that the road surface causes as the external forces release [7].

While designing automobile suspension system, usually the comfort that is taken by passengers is the first priority to be considered and the properties of grounding shall be considered as well, but the stroke of cushion only needs to accord with the large operative stroke [8]. A lot of study that apply control methods to the design of automobile suspension system, but need to have accurate data of the system parameters and signals of disturbance from road surface first, otherwise the robust of the system will be very weak [8]. Researches have proved that fuzzy preview control method can improve the properties of grounding effectively, but have not mentioned the comfort of taking[3]. The Tunable Fuzzy Logic Control applied on the automobile suspension systems the better comfort of taking can be obtained, but
because its specific limitation makes the state feedback of signals of disturbance unable to measure. The automobile suspension systems is in order to reduce the disturbance and vibration of external world to the automobile, that makes the behavior of the system approaching closely to the reference model that designer preset [4]. So, no matter which kind of estimating device is adopted to do the parameter estimation, all unable to give consideration to comfortableness and manipulation, and an extra brake component, the necessary expenses are very high.

An automobile suspension system fine or not depends on the design of the system parameters, and corresponding road surface situations. the kinematic model and time response of the body of car vary with the changes of the speed of the automobile, system parameters and road surface situations. Therefore, apply computerized method to simulate the kinmonic status and responses of the car body with respect to different parameters of suspension systems and road surface situations, can really save the huge expenses in experiments, and obtained the valuable design basis economically.

2. Mathematical Model

2.1. Explanation of Parameters and symbols

The parameters set in this paper are shown as in Fig. 1, its relevant explanation of symbols is as follows:

\(a(t)\): The function of the slope variation of road surface

\(b\): Damping coefficient of the damping device

\(D\): Diameter of the spring coil

\(d\): Diameter of the spring wire

\(G\): Transverse modulus of elasticity of the spring

\(k\): Modulus of elasticity of the spring

\(L\): The interval between shaft of front wheel and rear wheel
$l_1$: Vertical distance between front wheel and center of mass

$l_2$: Vertical distance between rear wheel and center of mass

$m$: Total mass of the automobile

$n$: Number of active coils

$p$: Distributive weight of front wheel

$q$: Distributive weight of rear wheel

$r$: Radius of turn round

$T$: Vibrations while advancing

$t$: Time of simulation

$u_1$: Position component of front wheel in vertical direction

$u_1'$: Velocity component of front wheel in vertical direction

$u_2$: Position component of rear wheel in vertical direction

$u_2'$: Velocity component of rear wheel in vertical direction

$v_o$: Initial velocity of the automobile

$y_c$: Position component of the center of mass in vertical direction

$y_c'$: Velocity component of the center of mass in vertical direction

$\theta$: Angular position of the center of mass

$\theta'$: Angular velocity of the center of mass

\[ k b b_k \]

\[ m J \]

\[ y_{11} \]

\[ y_{21} \]

\[ u_1 \]

\[ u_2 \]

\[ k \]

\[ b \]

\[ t=0 \]

\[ v_0 \]

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**Fig. 1 Schematic diagram of the mobile suspension system**
2.2. Derivation of the mathematic model

In the condition that both consider linear and angular motion of an automobile, the two equations can be gotten as follows:

\[ m\ddot{y}_c = 2k(u_1 - y_1) + 2b(u_1' - y_1') + 2k(u_2 - y_2) + 2b(u_2' - y_2') \quad (1) \]

\[ J\ddot{\theta} = -l_2 \left[ 2k(u_2 - y_2) + 2b(u_2' - y_2') \right] + l_1 \left[ 2k(u_1 - y_1) + 2b(u_1' - y_1') \right] \quad (2) \]

When \( \theta \) is very small, through the coordinate transformation we have:

\[ y_1 = y_c + l_1\theta \quad (3) \]

And

\[ y_2 = y_c - l_2\theta \quad (4) \]

Differentiating Eq. (3) and Eq. (4) with respect to time, we have:

\[ y_1' = y_c' + l_1\theta' \quad (5) \]

And

\[ y_2' = y_c' - l_2\theta' \quad (6) \]

Substituting Eq. (3)-(6) into Eq. (1)-(2), then we have:

\[ m\ddot{y}_c = -4by_c' - 4ky_c + 2b(l_2 - l_1)\theta' + 2k(l_2 - l_1)\theta + 2k(u_1 + u_2) + 2b(u_1' + u_2') \quad (7) \]

\[ J\ddot{\theta} = (l_1 - l_2)2by_c' + 2k(l_2 - l_1)y_c - 2b(l_1^2 + l_2^2)\theta' - 2k(l_1^2 + l_2^2)\theta \\
- 2b(l_2u_2' - l_1u_1') - 2k(l_2u_2 - l_1u_1) \quad (8) \]

Making definitions of relevant variables as follows:

\[
\begin{align*}
y_c &= x_1 \\
y_c' &= x_1' = x_2 \\
y_c'' &= x_1'' = x_2' \\
\theta &= x_3 \\
\theta' &= x_3' = x_4 \\
\theta'' &= x_3'' = x_4' \quad (9)
\end{align*}
\]
Substituting Eq.(9)-(10) into Eq.(7)-(8), after rearranging we have:

\[ x_2' = \frac{1}{m} \left[ -4kx_1 - 4bx_2 + 2k(l_2 - l_1)x_3 + 2b(l_2 - l_1)x_4 + 2ku_1 + 2ku_2 + 2bu_1' + 2bu_2' \right] \]

\[ x_4' = \frac{1}{J} \left[ 2k(l_2 - l_1)x_1 + 2b(l_2 - l_1)x_2 - 2k(l_1^2 + l_2^2)x_3 - 2b(l_1^2 + l_2^2)x_4 + 2kl_1u_1 - 2kl_2u_2 + 2b(l_1' + l_2') \right] \]

Expressing above-mentioned equations by matrix, we have:

\[
\begin{bmatrix}
    x_1' \\
    x_2' \\
    x_3' \\
    x_4'
\end{bmatrix} =
\begin{bmatrix}
    0 & 1 & 0 & 0 \\
    -4k & -4b & 2k(l_2 - l_1) & 2b(l_2 - l_1) \\
    m & m & m & m \\
    2k(l_2 - l_1) & 2b(l_2 - l_1) & -2k(l_1^2 + l_2^2) & -2b(l_1^2 + l_2^2) \\
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4
\end{bmatrix} +
\begin{bmatrix}
    0 & 0 & 0 & 0 \\
    2k & 2b & 2k & 2b \\
    m & m & m & m \\
    2kl_1 & 2bl_1 & -2kl_2 & -2bl_2 \\
\end{bmatrix}
\begin{bmatrix}
    u_1' \\
    u_1 \\
    u_2' \\
    u_2
\end{bmatrix}
\]

2. Numerical simulation

Several domestic brand automobiles with 1.6 liters of displacement are taken as examples of simulation in this approach. The relevant average data are shown as follows:

\[ m_{\text{average}} = 1880 \text{ kg} \]

\[ p = 65 \% \]

\[ q = 35 \% \]

\[ L = 2.55 \text{ m} \]

\[ r = 1.315 \text{ m} \]

\[ l_i = L \times p = 2.55 \times 0.65 = 1.6575 \text{ m} \]
\[ l_2 = L \times q = 2.55 \times 0.35 = 0.8925 \text{m} \]

\[ J = m \times r^2 = 3250 \text{ kg/m}^2 \]

\[ G = 8 \times 10^3 \text{ kg/mm}^2 \]

\[ D = 13 \text{ mm} \]

\[ n = 3.14 \text{ mm} \]

\[ d = 175 \text{ mm} \]

\[ k = \frac{G \times D^4}{8 \times n \times d^3} \text{ kgf/mm} \pm 20\% \]

\[ k : b = 3 : 1 \]

\[ k_{\text{max}} = 3840 \text{ kgf/mm} \]

\[ k_{\text{average}} = 3200 \text{ kgf/mm} \]

\[ k_{\text{min}} = 2560 \text{ kgf/mm} \]

\[ b_{\text{max}} = 1280 \text{ kgf} \]

\[ b_{\text{average}} = 1066 \text{ kgf} \]

\[ b_{\text{min}} = 853 \text{ kgf} \]

\[ v_0 = 10 \text{ m/s} \]

\[ t = 10 \rightarrow 20 \text{ sec} \]

### 4. Results and discussions

#### 4.1. Influence of damping coefficient to the system responses

We assume that an automobile moves with a constant speed \( v_0 = 10 \text{ m/s} \) and the modulus of elasticity of the spring \( k_{\text{average}} = 3200 \text{ kgf/mm} \). The simulation of the automobile motion systems responses is implemented with respect to different
damping coefficient, as the function of slope variation of road surface $a = \sin t$. The results of simulation are shown in Fig.2.

According to the results shown in Fig.2(a), the velocity component of the center of mass in vertical direction and its maximum overshoot reduce as damping coefficient increases, But there is no obvious difference to the steady time of system under different damping coefficient.

According to the results shown in Fig.2(b), the angular velocity of the center of mass becomes lower as damping coefficient increases, at this time, Not only the maximum overshoot is smaller, and steady time is also shorter.
Fig. 2 Magnitude plot of the mobile suspension system of the with the $v_0 = 10$ m/s

and $k_{\text{average}} = 3200$ kgf/mm, 当 $a = \sin t$ 時系統之響應

The results of simulation are shown in Fig.3 (a) as the function of slope variation of road surface $a = |\sin t|$. According to the results shown in Fig.3(a), the velocity component of the center of mass in vertical direction and its maximum overshoot also reduce as damping coefficient increases, and the steady time do not have obvious difference comparing with results as $a = \sin t$. Moreover, from the results as shown in Fig.3(a), the angular velocity of the center of mass becomes much lower as damping coefficient increases, but the steady time is not to be affected.
In terms of discussions mentioned above about results as shown in Fig. 2 and
Fig. 3, the conclusions can be made as follows:

(1). The velocity component of the center of mass in vertical direction and the angular velocity of the center of mass can be reduced effectively, and lower the maximum overshoot of the system as the damping coefficient increases.

(2). The influence is not obvious to the steady time of the system response as the damping coefficient varies.

4.2. Influence of the modulus of elasticity of the spring to the system responses

We assume that an automobile moves with a constant speed $v_0 = 10 \text{ m/s}$ and damping coefficient $b_{\text{average}} = 1066 \text{ kgf}$. The simulation of the automobile motion systems responses is implemented with respect to different the modulus of elasticity of the spring as the function of slope variation of road surface $a = \sin t$. The results of simulation are shown in Fig. 4.

According to the results shown in Fig. 4(a), the angular velocity of the center of mass and the maximum overshoot increase as the modulus of elasticity of the spring increases, at this time, but the rising time is shorter.

According to the results shown in Fig. 4(b), the response time of the angular velocity of the center of mass is shorter, and quickly reaches the steady state as the modulus of elasticity of the spring increases.
The results of simulation are shown in Fig.5 as the function of slope variation of road surface $a = |\sin t|$. According to the results shown in Fig.5(a), the
maximum overshoot obviously becomes larger, and the response time becomes shorter as the modulus of elasticity of the spring increases. Moreover, from the results as shown in Fig.5(b), the angular velocity of the center of mass becomes higher and the response quickly, but the steady time is not to be affected as damping coefficient increases.
Basing on discussions about the results as shown in Fig.4 and Fig.5, the conclusions can be made as follows:

(1). The velocity component of the center of mass in vertical direction and the angular velocity of the center of mass will increase as the damping coefficient increases.

(2). improving the system response time and lowering steady time can be reached by means of increasing damping coefficient.

5. Conclusions

The design of an automobile suspension system does affect its comfortableness and manipulation, the proper selection of the spring and damping device is an important subject to it. To construct the automobile kinematic model based on the kinematics and dynamics, to do the qualitative analysis and simulation with respect to different modulus of elasticity of the spring and damping coefficient by computer programs and to discuss the influence to the motion of vehicles by both of them have done in this approach. An effective and economic analytical method and the valuable
design basis are also provided for designing the automobile suspension systems.

References

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