Implementing Discrete Dynamic Integral Sliding Surface Control to Hydraulic Position Control

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ABSTRACT

The Variable Structure Control system is a great robust control method. However, its classic existence condition seems not enough when applied in discrete systems. Due to the existence of the sampling interval in discrete systems, the controller can not correct the system’s response real-time during the interval. The small switching gains are necessary in order not to be divergent to the switching surface. However, to quickly reach the sliding surface, the large switching gains have to be chosen. In this paper, based on the different viewpoint can develop the additional restriction. It does not only offer the definitions of ranges of the switching gains but also their domains. The Discrete Dynamic Integral Sliding Surface Control (DDISSC) can solve these problems. It can not only reduce the time to reaching the sliding surface, but also pay less control effort to achieve this surface. Finally, the hydraulic position servo control system is used to demonstrate the potential of the proposed control method. CPLD based control board will implement the proposed rule by full digital technology. Simulation and experimental results will show the robustness to against the parameters' variation and external disturbance.

Keyword : Discrete Dynamic Integral Sliding Surface Control (DDISSC), Variable Structure Control, dynamic sliding surface, Hydraulic Position Control.
1 Introduction

Since Dr. Utkin proposed the Variable Structure Control (VSC) system [1], it has been widely studied and applied to many practical systems. On the basis of the pole-placement method, one can previously design the sliding surface. Once the system’s sliding motion is locked on this surface, the dynamic response will be robust to against the parameter variations. At the same time, the system's order will be reduced at least one order. Hence, the stable control rule is developed according to the existence condition of the sliding mode, that is $s(t)s(t) < 0$ where $s(t)$ means the variable of sliding motion in continuous systems. However, this condition is not enough in the discrete systems discussed by Dr. Furuta [2]. Converting it to discrete form, that is $s(k)[s(k + 1) - s(k)] < 0$ (1)

will cause the sliding motion toward the siding surface. It can not make sure that the motion is convergent to this surface. The additional restriction is introduced to make the sliding motion convergent [2-3], that is $|s(k + 1)| < |s(k)|$ (2)

It provides a good ideal to design stable discrete sliding motion based on the viewpoint of scalar. This paper will discuss the existence conditions by the geographical viewpoint [4-6]. The motion restriction is used to guarantee the convergence, that is $|s(k + 1) - s(k)| < \frac{\xi}{2}$ (3)

where $\xi$ is a small positive constant. It causes the sliding motion converge into a small range $|s(k)| \leq \xi$ called ‘non-ideal’ discrete sliding motion. This paper will show that the existence conditions will not only offer the choosing ranges but also decide the domains related the sampling time interval.

In some discussions, the VSC is weak in handling the external disturbance except the usage of large constant switching gains. But the large constant switching gains usually cause the heavy chattering. This paper adds one integral controller into the VSC to be the Integral VSC (IVSC) [4-6]. By re-arranged the form of the sliding surface, the IVSC can own the advantage of both controllers. Besides, the system’s dynamic controlled by the sliding surface is only when the sliding motion is locked on the surface. To reach the surface early, large switching gains are the best way. However, the
control signal is changed only in the sampling time in discrete systems. The large gains will cause the large chattering. Even more, the dynamic response may become to be unstable during the sampling interval. This paper proposes the dynamic sliding surface, first order of difference equation of sliding variable, to solve this problem, that is

\[ s(k + 1) = -ps(k) + q \text{sgn}[s(k)] , \quad s(0) = \sum_{i=1}^{n} a_i x_i(0) \]  

(4)

where \( p \) and \( q \) are chosen to satisfy the Jury’s test. Because the initial value of the dynamic sliding surface is consisted of the system’s initial values, the controllers do not need to pay much effort to fix on this surface. The linear controller, called equivalent controller, will take the motion to track this dynamic sliding surface. The non-linear controller, called switching part, will make the motion toward and convergent to the surface. The Discrete Dynamic Integral Sliding Surface Control (DDISSC) method is not only overcome the variation in system’s parameters but also eliminate the effect from external disturbance.

Furthermore, this paper applies the DDISSC method to hydraulic position servo control systems [4-8]. With the great advance in microprocessors, the full digital controller has become the trend in the future. The newest idea is to collect all function in a chip, that is systems on chip (SOC). This paper will implement the proposed control rule by CPLD. The control board will increase the calculating speed and reduce the volume of hardware. Simulation results will demonstrate the potential of the proposed method. Experimental results will show the practical capability in industrial applications.

2 Discrete dynamic integral sliding surface control

2.1 Design of control function

Let the studied continuous system’s equation be

\[ \dot{x}_i = x_{i+1}, \quad i = 1, \ldots, n-1 \]  

(5a)

\[ \dot{x}_n = -\sum_{i=1}^{n} a_i x_i + bu + f_L \]  

(5b)

Here, the integral controller is arranged as

\[ \dot{z} = r - x_1 \]  

(6)
where $r$ is the set point. In order to discuss in discrete form, the differential equations have to be transferred to difference equations by forward difference, that is
\[
X = \frac{X(k+1) - X(k)}{T},
\]
where $T$ is the sampling time interval. Then, the discrete forms of eqs. (5-6) become to be
\[
z(k+1) = z(k) + Tr(k) - x_i(k)]
\]
(7a)
\[
x_i(k+1) = x_i(k) + T x_{i+1}(k) \quad i = 1, \ldots, n - 1
\]
(7b)
\[
x_n(k+1) = x_n(k) + T \left[ - \sum_{i=1}^{n} a_i x_i(k) + bu(k) + f_L(k) \right]
\]
(7c)
Choose the sliding surface as
\[
\left\{s(x_i(k)) \right\} = c_i [x_i(k) - K_i z(k)] + \sum c_i x_i(k) = 0
\]
(8)
The dynamic of the sliding surface is designed to be
\[
s(k+1) = -ps(k) + q sgn[s(k)]
\]
(9)
The control function is divided into two parts, that is
\[
u(k) = u_{eq}(k) + u_s(k)
\]
One is the equivalent controller, and the other one is the switching controller. The equivalent controller of control function is figured out when the parameters are in their nominal values and no load existed. The switching controller is used to eliminate the affection of parameter's variation and disturbance. Once, the sliding motion is locked on the dynamic sliding surface, that is
\[
\left[ Tc_a a_1 + Tc_1 K_f + c_1 + pc_1 \right] x_1(k) + \sum_{i=2}^{n} \left[ Tc_a a_i + Tc_{i-1} + c_i + pc_i \right] x_i(k) + \sum_{i=2}^{n} \left[ Tc_o a_i^0 - Tc_{i-1} - c_i - pc_i \right] x_i(k) + c_1 K_f i z(k) - Tc_1 K_f r(k) - q sgn[s(k)] + Tc_o bu(k) + Tc_o f_L(k) = 0
\]
(10)
Then, the equivalent controller is
\[
u_{eq}(k) = \frac{1}{Tb_o c_o} \left\{ \left[ Tc_o a_i^0 - Tc_1 K_f - c_1 - pc_1 \right] x_i(k) + \sum_{i=2}^{n} \left[ Tc_o a_i^0 - Tc_{i-1} - c_i - pc_i \right] x_i(k) + \sum_{i=2}^{n} \left[ Tc_o a_i^0 - Tc_{i-1} - c_i - pc_i \right] x_i(k) + c_1 K_f + pc_1 K_f \right\}
\]
(11)
where \( a_i^0, b^0 \) and \( \Delta a_i, \Delta b \) are the nominal values and variation of \( a_i \) and \( b \), respectively, that is

\[
a_i = a_i^0 + \Delta a_i, \quad i = 1, \ldots, n
\]

\[
b = b^0 + \Delta b
\]

(12a)

(12b)

Let the switching controller is based on state switching, that is

\[
u_s(k) = \varphi_i(k) [x_i(k) - K_i z(k)] + \sum_{i=2}^n \varphi_i(k)x_i(k)
\]

(13)

where \( \varphi_i \cdots \varphi_n \) are switching gains and decided by the existence conditions.

Substituting eqs (11-13) into the difference of the sliding surface, one can have

\[
s(k + 1) - s(k) =
\]

\[
- pc_i \left[ 1 + \frac{\Delta b}{b^0} \right] - c_i \frac{\Delta b}{b^0} - Tc_n \Delta a_i + T \frac{\Delta b}{b^0} c_i a_i^0 - T \frac{\Delta b}{b^0} c_i K_i + Tc_n b \varphi_i(k) \left[ x_i(k) - K_i z(k) \right]
\]

\[
+ \sum_{i=2}^n \left[ - pc_i \left[ 1 + \frac{\Delta b}{b^0} \right] - c_i \frac{\Delta b}{b^0} - Tc_n \Delta a_i + T \frac{\Delta b}{b^0} c_i a_i^0 - T \frac{\Delta b}{b^0} c_i K_i + Tc_n b \varphi_i(k) \right] x_i(k)
\]

\[
+ \left[ F(k) + Tc_n b \varphi_{n+1}(k) \right]
\]

(14)

where

\[
F(k) = T \frac{\Delta b}{b^0} c_i a_i^0 K_i z(k) - T \frac{\Delta b}{b^0} c_i K_i^2 - Tc_n \Delta a_i K_i z(k) + Tc_n f_k(k)
\]

\[
+ T \frac{\Delta b}{b^0} c_i K_i r(k) - \left( 1 + \frac{\Delta b}{b^0} \right) q \text{sgn}[s(k)]
\]

To satisfy the existence condition \([s(k + 1) - s(k)]s(k) < 0\) of sliding motion, according to eq (14) one has the stable ranges of the switching gains as

\[
\varphi_+ < \alpha_i = \inf \left\{ pc_i \left[ 1 + \frac{\Delta b}{b^0} \right] + c_i \frac{\Delta b}{b^0} + Tc_n \Delta a_i - T \frac{\Delta b}{b^0} c_i a_i^0 + T \frac{\Delta b}{b^0} c_i K_i \right\} / Tc_n b
\]

\[
\varphi_- > \beta_i = \sup \left\{ pc_i \left[ 1 + \frac{\Delta b}{b^0} \right] + c_i \frac{\Delta b}{b^0} + Tc_n \Delta a_i - T \frac{\Delta b}{b^0} c_i a_i^0 + T \frac{\Delta b}{b^0} c_i K_i \right\} / Tc_n b
\]

\[
\varphi_+ < \alpha_i = \inf \left\{ pc_i \left[ 1 + \frac{\Delta b}{b^0} \right] + c_i \frac{\Delta b}{b^0} + Tc_n \Delta a_i - T \frac{\Delta b}{b^0} c_i a_i^0 + T \frac{\Delta b}{b^0} c_i K_i \right\} / Tc_n b
\]

\[
\varphi_- > \beta_i = \sup \left\{ pc_i \left[ 1 + \frac{\Delta b}{b^0} \right] + c_i \frac{\Delta b}{b^0} + Tc_n \Delta a_i - T \frac{\Delta b}{b^0} c_i a_i^0 + T \frac{\Delta b}{b^0} c_i K_i \right\} / Tc_n b
\]

\[
\varphi_+ < \alpha_i = \inf \left\{ pc_i \left[ 1 + \frac{\Delta b}{b^0} \right] + c_i \frac{\Delta b}{b^0} + Tc_n \Delta a_i - T \frac{\Delta b}{b^0} c_i a_i^0 + T \frac{\Delta b}{b^0} c_i K_i \right\} / Tc_n b
\]

\[
\varphi_- > \beta_i = \sup \left\{ pc_i \left[ 1 + \frac{\Delta b}{b^0} \right] + c_i \frac{\Delta b}{b^0} + Tc_n \Delta a_i - T \frac{\Delta b}{b^0} c_i a_i^0 + T \frac{\Delta b}{b^0} c_i K_i \right\} / Tc_n b
\]
\[
\varphi_{n+1} = \begin{cases} 
\varphi_{n+1}^+ < \alpha_{n+1} = \inf \left\{ -F(k) \left/ Tc_n b \right. \right\} \\
\varphi_{n+1}^- > \beta_{n+1} = \sup \left\{ -F(k) \left/ Tc_n b \right. \right\}
\end{cases}
\]

Let \( \xi_1 + \cdots + \xi_{n+1} = \frac{\xi}{2} \). To satisfy this condition \(|s(k+1) - s(k)| < \frac{\xi}{2}\), one can obtain
\[
|s(k+1) - s(k)| \leq -pc_i \left[ 1 + \frac{\Delta b}{b^0} \right] - c_i \frac{\Delta b}{b^0} - Tc_n \Delta a_i + T \frac{\Delta b}{b^0} c_n a_i^0 - T \frac{\Delta b}{b^0} c_i K_i + Tc_n b \varphi_i (k) \left] x_i (k) - c_i + Tc_n b \varphi_i (k) \right] x_i (k) \\
+ \sum_{i=2}^n \left[ -pc_i \left( 1 + \frac{\Delta b}{b^0} \right) - c_i \frac{\Delta b}{b^0} - Tc_n \Delta a_i + T \frac{\Delta b}{b^0} c_n a_i^0 - T \frac{\Delta b}{b^0} c_i c_{i-1} + Tc_n b \varphi_i (k) \right] x_i (k) \\
+ \left[ F(k) + Tc_n b \varphi_{n+1} (k) \right] \leq [ \xi_1 + \cdots + \xi_{n+1} ]
\]

(15)

Then, the domains of the switching gains are
\[
L_i \leq \varphi_i \leq M_i, \quad i = 1, \cdots, n+1
\]

where
\[
M_i = \sup \left\{ \left[ pc_i \left( 1 + \frac{\Delta b}{b^0} \right) + c_i \frac{\Delta b}{b^0} + Tc_n \Delta a_i - T \frac{\Delta b}{b^0} c_n a_i^0 + T \frac{\Delta b}{b^0} c_i K_i \right] \left/ Tc_n b \right. \right\} + \sup \left\{ \xi_i \left/ x_i (k) - c_i + Tc_n b \varphi_i (k) \right. \right\}
\]
\[
L_i = \inf \left\{ \left[ pc_i \left( 1 + \frac{\Delta b}{b^0} \right) + c_i \frac{\Delta b}{b^0} + Tc_n \Delta a_i - T \frac{\Delta b}{b^0} c_n a_i^0 + T \frac{\Delta b}{b^0} c_i K_i \right] \left/ Tc_n b \right. \right\} - \sup \left\{ \xi_i \left/ x_i (k) - c_i + Tc_n b \varphi_i (k) \right. \right\}
\]
\[
M_i = \sup \left\{ \left[ pc_i \left( 1 + \frac{\Delta b}{b^0} \right) + c_i \frac{\Delta b}{b^0} + Tc_n \Delta a_i - T \frac{\Delta b}{b^0} c_n a_i^0 + T \frac{\Delta b}{b^0} c_i c_{i-1} \right] \left/ Tc_n b \right. \right\} + \sup \left\{ \xi_i \left/ x_i (k) \right. \right\}
\]
\[
L_i = \inf \left\{ \left[ pc_i \left( 1 + \frac{\Delta b}{b^0} \right) + c_i \frac{\Delta b}{b^0} + Tc_n \Delta a_i - T \frac{\Delta b}{b^0} c_n a_i^0 + T \frac{\Delta b}{b^0} c_i c_{i-1} \right] \left/ Tc_n b \right. \right\} - \sup \left\{ \xi_i \left/ x_i (k) \right. \right\}
\]
\[
M_{n+1} = \sup \left\{ -F(k) \left/ Tc_n b \right. \right\} + \sup \left\{ \frac{\xi_{n+1}}{T} \right\}
\]
\[
L_{n+1} = \inf \left\{ -F(k) \left/ Tc_n b \right. \right\} - \sup \left\{ \frac{\xi_{n+1}}{T} \right\}
\]

Let's take \( \varphi_{n+1} \) for example. The first condition decides the range. However, the second condition restricts the domain. Referring fig. 1, the width of available region of \( \varphi_{n+1} \) is
\[
\sup \left\{ \frac{\xi_{n+1}}{T} \right\}
\]
Obviously, in continuous systems the width will extend to be infinite.
because $T \to 0$, that is all real number. In discrete systems, the region will be related to the inverse of sampling time interval.

If one chooses the switching gains as $\varphi_i = -|\varphi_i^+| = |\varphi_i^-|$, then combining the domains and ranges of the switching gains, one obtains the choosing limitations defined as

$$\min\{|L_i|, |\alpha_i|\} \leq \varphi_i \leq \max\{|M_i|, |\beta_i|\}$$

Then the switching controller will be rewritten as

$$u_s(k) = \left[ \varphi_1(k)x_1(k) - K_1z(k) + \sum_{i=2}^{n} \varphi_i(k)x_i(k) \right] \text{sgn}(s(k))$$

(16)

Due to the existence of sign function of sliding surface, the switching controller will have the phenomena of chattering. It will make the implementation impossible. Hence, the function $M(s(k))$ will be used to smooth the control function. The function is arranged as

$$M(s(k)) = \frac{s(k)}{|s(k)| + \delta}$$

(17)

where $\delta$ is a small positive constant.

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2.2 Choose of dynamic sliding surface

Once, the sliding motion is exactly locked on the dynamic sliding surface, that is $s(k) = 0$. According to eq (8), the highest order variable can be replaced as the linear combination as
Substituting eq (18) into system equation (7), the reduced order system equation becomes to be

\[ z(k + 1) = z(k) + T[r(k) - x_i(k)] \]  
(19a)

\[ x_i(k + 1) = x_i(k) + T x_{i+1}(k), \quad i = 1, \ldots, n - 2 \]  
(19b)

\[ x_{n-1}(k + 1) = x_{n-1}(k) - T \left( \frac{c_i}{c_n} [x_i(k) - K_j z(k)] + \sum_{i=2}^{n} \left[ \frac{c_i}{c_n} x_i(k) \right] \right) \]  
(19c)

The characteristic equation of eq (19) is

\[
\left( \frac{Z - 1}{T} \right)^n + \frac{c_{n-1}}{c_n} \left( \frac{Z - 1}{T} \right)^{n-1} + \cdots + \frac{c_1}{c_n} \left( \frac{Z - 1}{T} \right) + \frac{c_i K_j}{c_n} = 0
\]  
(20)

where \( Z \) means the z-transformation operator. Then, according to the required response, one can decide the poles in continuous by pole-placement method, that is

\[
S^n + p_n S^{n-1} + \cdots + p_2 S + p_1 = 0
\]  
(21)

where \( S \) means the Laplace transformation operator. Due to the forward difference transformation, that is \( S = \frac{Z - 1}{T} \), one can obtain the parameters of the dynamic sliding surface by comparing two equations (20) and (21).

3 Dynamic model of electrohydraulic position servo systems

The block diagram of the hydraulic position servo system is shown in fig. 2.

![Block Diagram of Hydraulic Position Servo System](image)

Fig. 2 The block diagram of the hydraulic position servo system.

The valve displacement and the flow rate are governed by the orifice law (Chern, 1991, 1992; Merrit, 1982; Sohl, 1999), that is
\[ Q_L = X_s K_J \sqrt{P_s - \text{sgn}(X_s)P_L} = X_s K_s \]  \hspace{1cm} (22)

The \( K_J \) is a constant depended on the specific hydraulic component. The \( P_s \) and \( P_L \) are the supply pressure and the load pressure. Hence, the valve flow gain \( K_s \) will be depended on the working conditions. The volume and continuity expressions can be combined to yield

\[ Q_L = D \omega + C_v P_L + \frac{4\beta}{V_t} \dot{P}_L \]  \hspace{1cm} (23)

which is the usual form of the continuity equation. The \( D \) is volumetric displacement. The \( C_v \) is the total leakage coefficient. the \( \beta \) is the bulk modulus of the oil and the \( V_t \) is the total volume of the oil. The \( \omega \) is the velocity of the hydraulic cylinder. The resulting torque equation is

\[ T = DP_L = J\omega + B\omega + T_L \]  \hspace{1cm} (24)

where \( J \) is the total inertia coefficient of the hydraulic cylinder and \( B \) is the viscous damping constant. The spring load \( T_L \) will vary depending on the Hook's law. that is \( T_L = K_H \theta \) where \( K_H \) is the Hook's constant. The hydraulic cylinder position \( \theta \) is obtained by

\[ \dot{\theta} = 57.3\omega \]  \hspace{1cm} (25)

where the constant 57.3 cm/rad is the transforming gain from radius to centimeters. The variables of position \( \theta \), velocity \( \omega \) and load pressure \( P_L \), are all measurable.

4 Simulation and experimental results

4.1 Simulation results

Fig. 3 and Fig. 4 show the mathematical simulation results while applying the proposed control rule to the electrohydraulic servo system. The position response is shown in Fig.3. A constant load and a varying load proportioned to output position are added to the torque input. Comparing these responses in Fig. 3, the closeness of these lines shows the robust capability to against the external load. At the same time, the condition of varying in supply oil pressure is also simulated. The dynamic response is shown in Fig 3, too. Obviously, the proposed control rule can overcome the effect of the parameter variation, too. Fig. 4 shows the output control signal. Due to the dynamic
sliding surface, the chattering phenomenon of classical variable structure control has been eliminated. The smooth curve shows the practical capability of the proposed rule.

4.2 Experimental results

To demonstrate the performance of the proposed rule, experimental equipment is developed shown in Fig. 5. A PC-based controller is developed, including the encoder.
input and current output. The up/down counter will convert the signal from the encoder to position feedback. The control signal will be sent out from the current output of D/A converter to drive the servo valve. A spring is used to simulate the external load. The torque from the spring, which satisfied the Hook's law, will be proportioned to the position of the hydraulic cylinder. The control object is to maintain the advance position however the load from the spring is. Fig. 6 shows the responses of different external load existed under the same supply oil pressure. The lines are almost on the same position. It means the load will not affect the position output. Fig. 7 shows the responses of different external load existed and half oil pressure supplied. Comparing the dynamic between the varying and normal conditions, it is clearly that the proposed rule can practically overcome the affection from the load and parameter variation. The control signal under normal condition is shown in Fig. 8. Obviously, the control rule can be applied to the real industrial case.

Fig. 5 Experimental equipment.
Fig. 6 Experimental response of position.
(Normal condition and load existed conditions)

Fig. 7 Experimental response of position.
(Normal condition and load existed conditions with half supplied oil pressure)
5. Conclusion

The paper shows the detailed design procedure of dynamic integral sliding surface control. The dynamic sliding surface will reduce the control effort. At the same, it can efficiently minimize the reaching time. The simulation and experimental results show the practical capability of the proposed controller. Due to the widely application of hydraulic systems, this controller will be helpful in the further.

Reference


