Optimal Inspection Policy for a Lot-sizing Production System

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Abstract
The main goal of this research is to develop an inspection policy for the joint determination of both risk balance and homogeneous reliability of lot-sizing production system. The system of production process has a general deterioration distribution with increasing failure rate and non-self-announcing failures. Rather than develop a non-Markovian shock model, we focus on a quantile-based reliability model. This research will also provide a strategy of inspection based on the economic production quantity and examples of Weibull shock models be given to illustrate this inspection policy.

Keywords: inspection policy, production process, economic manufacturing quantity.

1. Introduction
Batch mode of production in most manufacturing industries is widely used because that in certain instances, the capacity to produce a part exceeds the part’s usage or demand rate. The classical economic manufacturing quantity model assumes that the output of the production system is defect-free. When developing economic manufacturing quantity (EMQ) models, consideration of controlling the quality of the product has generally not been taken into account. Optimal inspection policies for products with imperfect qualities were also investigated considerably. Rosenblatt and Lee (1986) have found that, when the production process is subject to a random shock that shifts the system from an in-control state to an out-of-control state followed by producing non-conforming items, the resulting optimal EMQ is smaller than that of the classical model as expected. Porteus (1986) has also observed similar results. For further review of the EMQ topics with imperfect production processes, the reader can refer to Ben-Daya (1998, 1999) and Sheu (2004). Suppose that failure of the production system can be detected only by inspection of product. And, inspection can not determine either the level of deterioration or the remaining life of the production system. Such is the case in many protective items such as circuit breakers and protective relays, as well as in spare or standby. If the production system is found to be failed by performing inspection, it is replaced immediately with an identical item and the production process need not to be broken down. Many studies, such as Rahim (1994), have effort on the similar production
system which is repairable. Uniform sampling intervals for Markovian shock models provide a constant integrated hazard over each interval. Rahim (1994), Being motivated by this, proposed that the length of sampling intervals should be chosen in such a way that the integrated hazard over each interval should be equal. Yang and Klutke (2000) were also deal inspection policies for maintaining deteriorating equipment with non-self-announcing failures. Their inspection policies utilize information from the inspection/repair history as well as the system lifetime distribution to schedule future inspections. If inspection, replacement and downtime cost are available, they also derived the expression for long-term expected cost. Our objectives are to present inspection policies and propose some solution procedures for simultaneous determination of the inspection intervals and the number of inspection in a production run. The reminder of this paper is organized as follows. In Section 2, we will describe the operation system and introduce both the notation and assumptions that are used in the proposed policies. Inspection policies are described in Section 3. In Section 4, we will discuss the relationship of policies and take numerical examples to demonstrate the policies of inspection in Section 5. The conclusion of this paper is shown in Section 6.

2. Inspection policy

Beginning with a new operation system, item in this system can be repaired to the bad-as-old condition at the first inspection age at which it is found failed. The time to repair the failed item can be negligible and we assume that this operation system will not be interrupted by inspection or repair. An inspection policy consists of a sequence of inspection ages \{x_1, x_2, \cdots\} with \(0 = x_0 < x_1 < x_2 < \cdots\) and the \(i\)th inter-inspection age can be defined as \(h_i = x_i - x_{i-1}\). There are four inspection policies to be introduced for different scenarios.

2.1. The quantile-based reliability in a production system

The most widely used but weak inspection policy is perhaps constant inter-inspection. It is to schedule inspections periodically with the advantage of being simple to implement and relatively easy to analyze. Such a policy has constant inter-inspection \(h_i = h\), for all \(i\), and inspection ages \(\{h, 2h, \cdots\}\). The periodic inspection policy tends to over-inspect at less likely failure times and under-inspect at more likely failure times. Thus, it employs no information about the remaining life that is inherent in the sequence of previous inspection times. Yang and Klutke (2000) proposed an inspection policy QBI(\(\alpha_0\)) with quantile-based reliability. Let the sequence of inspection ages be \(\{x_1, x_2, \cdots\}\) and we denote \(t\) as the time to failure. And, \(t\) can be expressed as a random variable with a continuous, strictly increasing distribution function \(F(t)\). The quantile-based reliability \(\alpha_0\), \(0 \leq \alpha_0 \leq 1\), is fixed and known. Then, the inspection ages are

\[
x_i = \sup \{x > x_{i-1} : P\{t > x \mid t > x_{i-1}\} \geq \alpha_0\}, \quad \text{for } i = 2, 3, \cdots.
\]

(1)
Thus,
\[
x_i = F^{-1}(\alpha'_i), \text{ for } i = 1,2,\ldots.
\] (3)

It is trivial that \( h_{i+1} \leq h_i \), for \( i = 1,2,\ldots \). In other words, the inter-inspection times \( h_i, h_2, \ldots \) are non-increasing, i.e. \( h_i \geq h_{i+1} \). The periodic inspection policy is a special case when \( h_1 = h_2 = \cdots = h \). For lot-sizing production system, the cycle time of operation system \( L \) can be expressed as
\[
L = \sum_{i=1}^{m} h_i = F^{-1}(\alpha'_m),
\] (4)

where \( m \) is the known number of inspection in the cycle time of operation system. We denote such a policy as QBI(\( \alpha_0, m \)).

2.2. The balance of risk under fixed economic manufacturing quantity

The quality and the quantity of the products are equally important in many industrial situations. Classical optimal manufacturing quantity is usually determined by trading off the set-up costs and the inventory holding costs. For a Markovian shock model, a uniform inspection scheme provides a constant integrated hazard rate over each interval when the number of inspection \( m \) is determined. And such a policy has constant inter-inspection time \( h = L/m \) and inspection ages \( \{h, 2h, \cdots, mh\} \). Being motivated by this, Rahim (1994) extended the idea to Non-Markovian shock models and proposed that the inter-inspection times \( h_i = x_i - x_{i-1} \) should be chosen in such a way that the integrated hazard over each interval should be equal, i.e.:
\[
\int_{0}^{x_i} r(t) \, dt = \frac{1}{m} \int_{0}^{L} r(t) \, dt,
\] (5)

and
\[
\int_{x_{i-1}}^{x_i} r(t) \, dt = \int_{0}^{x_i} r(t) \, dt, \quad i = 1, \cdots, m,
\] (6)

where \( \int_{a}^{b} r(t) \, dt \) represents the expected failure number over interval \( [a,b] \) and \( x_0 = 0 \).

Keeping a constant integrated hazard over each sampling interval is equivalent to stating that the probability of a shift in an interval, given no shift until its start, is a constant for all intervals. The deterioration of production systems is inherent to most manufacturing industries. If the distribution of the time to failure is increasing failure rate, the inter-inspection times \( h_1, h_2, \cdots, h_m \) are also non-increasing, as expected. According to the lot-sizing production cycle time \( L \) and the number of inspection \( m \), we have the inspection ages \( \{x_1, x_2, \cdots, x_m\} \):
\[
x_i = \sum_{j=1}^{i} h_j, \quad i = 1, \cdots, m.
\] (7)

We denote such a policy of risk balance as RB(\( L, m \)).
2.3. The hybrid inspection policy

For most manufacturing industries, the quantile-based reliability is a specific aim for standard operation process. And, it is an unknown but important objective to determine the optimal number of inspection and the inspection ages \( \{x_1, x_2, \cdots, x_m\} \) during each production run. When the cycle time \( L \) is given and under the known quantile-based reliability \( \alpha_0 \), we have the initial inspection ages from Eq. (3): \( x_i = F^{-1}(\alpha'_0) \), for \( i = 1, 2, \cdots \). And,

\[
\sum_{i=1}^{m-1} h_i < x_{m-1} \leq x_m = \sum_{i=1}^{m} h_i .
\] (8)

Thus, we could determine the number of inspection \( m \) from (8) and the inspection ages \( \{y_1, y_2, \cdots, y_m\} \) can be generated by policy RB(\( L, m \)).

\[
\int_{y_{i-1}}^{y_i} r(t) \, dt = \int_{0}^{y_i} r(t) \, dt = \frac{1}{m} \int_{0}^{L} r(t) \, dt , \quad i = 1, \cdots, m,
\] (9)

where \( y_0 \equiv 0 \). We denote such a hybrid policy as H1(\( \alpha_0, L \)).

If the cycle time \( L \) is unknown, the realistic situation could be according to the known quantile-based reliability \( \alpha_0 \) and the lower bound of inter-inspection age \( h_{LB} \). The number of inspection \( m \) could be also determined by the following relationships:

\[
x_i = F^{-1}(\alpha'_0) \quad \text{and} \quad h_m = \inf\{x_i - x_{i-1} : x_i - x_{i-1} \geq h_{LB}\}
\] (10)

Let \( y_m = L = \sum_{i=1}^{m} h_i \leq x_m \) and the inspection ages \( \{y_1, y_2, \cdots, y_m\} \) can be similarly generated by policy RB(\( L, m \)) as Eq.(9). We denote such a hybrid policy as H2(\( \alpha_0, h_{LB} \)).

3. Discussion

3.1. Equivalence between QBI(\( \alpha_0, m \)) and RB(\( L, m \))

The balance of risk is stressed under the RB(\( L, m \)) inspection policy. While the production cycle time \( L \) and the number of inspection \( m \) are predetermined, we can use equations (5) and (6) to obtain the inspection ages \( \{x_1, x_2, \cdots, x_m\} \). At the first inspection age \( x_1 \), the reliability \( R(x_1) \) can be expressed as

\[
P\{t > x_1\} = R(x_1) = \exp\{-\int_{0}^{x_1} r(t) \, dt\} .
\] (11)

It implies that

\[
x_1 = R^{-1}(\exp\{-\int_{0}^{x_1} r(t) \, dt\}) = F^{-1}(\alpha'_0) ,
\] (12)

where \( \alpha_0 \) is the quantile-based reliability. And, at the ith inspection age:
\[ x_i = \sum_{j=1}^{i} h_j, i = 2, \ldots, m, \]

we have

\[ P\{t > x_i\} = R(x_i) = \exp\{-\int_0^{x_i} r(t) \, dt\} = \exp\{-\sum_{j=1}^{i} \int_{x_{j-1}}^{x_j} r(t) \, dt\} \]. \tag{13}

From Eq.(6):

\[ P\{t > x_i\} = \exp\{-i \cdot \int_0^{x_i} r(t) \, dt\} = (\exp\{-\int_0^{x_i} r(t) \, dt\})^i = \alpha_0^i \] \tag{14}

It implies that

\[ x_i = F^{-1}(\alpha_0^i) \quad \text{for} \quad i = 1, \ldots, m. \]

Because that the production cycle time \( L \) has been predetermined, the reliability is also determined (i.e. \( \text{QBI}(\alpha_0, m) \)) at the same time under fixed number of inspection \( m \). On the other hand, if the quantile-based reliability has been predetermined, we have also the same sequence of inspection ages with \( \text{RB}(L, m) \) under fixed number of inspection \( m \).

3.2. The domination of hybrid inspection policy

Fixed lot-sizing batch mode of production and constant production rate could be anticipated from economic manufacturing quantity or the characteristic of batch mode in most industrial. Thus, we have the known production cycle time \( L \) for each production run. In a complete production run, the number of inspection \( m \), an unknown but important objective to determine, may also be determined. Under policy \( \text{QBI}(\alpha_0, m) \), we have the inspection ages from Eq.(3):

\[ x_i = F^{-1}(\alpha_0^i), \quad \text{for} \quad i = 1, \ldots, m, \]

with reliability:

\[ R(x_i) = \exp\{-\int_0^{x_i} r(t) \, dt\}, \quad \text{for} \quad i = 1, \ldots, m. \]

Under policy \( \text{H1}(\alpha_0, L) \) and \( \text{H2}(\alpha_0, h_{LB}) \) as described in Section 2.3, we have the inspection ages from Eq.(8) to Eq.(10):

\[ \int_0^{y_i} r(t) \, dt = \frac{i}{m} \cdot \int_0^{L} r(t) \, dt \leq \frac{i}{m} \cdot \int_0^{x_i} r(t) \, dt = \int_0^{x_i} r(t) \, dt \]

Because of \( y_i \leq x_i \), the ratio \( \frac{R(y_i)}{R(x_i)} = \exp\{\int_0^{y_i} r(t) \, dt\} \geq e^0 = 1 \), i.e. \( \alpha = R(y_i) \geq R(x_i) = \alpha_0 \).

We could improve the reliability from \( \alpha_0 \) to \( \alpha \) by adopt \( \text{H1}(\alpha_0, L) \) or \( \text{H2}(\alpha_0, h_{LB}) \).

4. Example

We present numerical examples to demonstrate the applications under non-Markovian shock models. Recall that the Weibull distribution function with scale parameter \( \lambda \) and shape
parameter $\beta$ is given by $F(t) = 1 - \exp\{- (\lambda t)^\beta\}$ for $t > 0$. The corresponding failure rate is $r(t) = (\lambda^\beta \cdot (\lambda t)^{\beta-1}$ for $t > 0$. Then, the inspection ages $\{x_1, x_2, \cdots\}$ and inter-inspection times $h_i = x_i - x_{i-1}$ can be determined from Eq.(3) as follows.

$$x_i = h_i = F^{-1}(\alpha_o) = \frac{1}{\lambda} \left[ -\ln(\alpha_o) \right]^{1/\beta}, \quad x_i = \frac{1}{\lambda} \left[ -i \cdot \ln(\alpha_o) \right]^{1/\beta} = i^{1/\beta} \cdot x_i, \quad h_i = [i^{1/\beta} - (i-1)^{1/\beta}] \cdot h_i.$$  

For table 1, we use $\alpha_o = 0.99$ and $\lambda = 0.05$, $\beta = 2.00$. The first inspection age is 2.005 with reliability 0.990. Because of linearly increasing failure rate, the inter-inspections are decreasing. The reliability is larger than 0.9 before the first 10 inspections. If we qualify the reliability as 0.9, the production cycle time should not be more than 6.340 and the number of inspection is 10. In the other words, the corresponding inspection policy of risk balance is RB(6.340, 10). We have the same inspection ages and inter-inspection by Eq.(5) and Eq.(6).

In the above example, suppose the production cycle time 5 is predetermined and the quantile-based reliability should be larger than 0.99. The number of inspection can be determined by table 1 and the reliability at the 7th inspection is 0.932. Table 2 shows the hybrid inspection policy $H1(0.99, 5)$.

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$x_i$</td>
<td>2.005</td>
<td>2.836</td>
<td>3.473</td>
<td>4.010</td>
<td>4.483</td>
<td>4.911</td>
<td>5.305</td>
<td>5.671</td>
<td>6.015</td>
<td>6.340</td>
<td>...</td>
</tr>
<tr>
<td>$h_i$</td>
<td>2.005</td>
<td>0.831</td>
<td>0.637</td>
<td>0.537</td>
<td>0.473</td>
<td>0.428</td>
<td>0.394</td>
<td>0.366</td>
<td>0.344</td>
<td>0.325</td>
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</tr>
<tr>
<td>$R(x_i)$</td>
<td>0.990</td>
<td>0.980</td>
<td>0.970</td>
<td>0.961</td>
<td>0.951</td>
<td>0.941</td>
<td>0.932</td>
<td>0.923</td>
<td>0.914</td>
<td>0.904</td>
<td>...</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i$</td>
<td>1.890</td>
<td>2.673</td>
<td>3.273</td>
<td>3.780</td>
<td>4.226</td>
<td>4.629</td>
</tr>
<tr>
<td>$h_i$</td>
<td>1.890</td>
<td>0.783</td>
<td>0.601</td>
<td>0.506</td>
<td>0.446</td>
<td>0.403</td>
</tr>
<tr>
<td>$R(x_i)$</td>
<td>0.991</td>
<td>0.982</td>
<td>0.974</td>
<td>0.965</td>
<td>0.956</td>
<td>0.948</td>
</tr>
</tbody>
</table>

Under such policy, the inter-inspections are also decreasing and the inspection ages advance against policy QBI resulting in higher reliability. The reliability at the 7th inspection is 0.939 which is higher than 0.932 in table 1, for example. Because of the final inspection age is controlled at the end of production cycle time, the efficiency of inspection is sufficiently developed.

5. Conclusion

Some inspection policies for deteriorating lot-sizing production system subject to a non-Markovian random shock are introduced, modified and compared. These policies are
simple to schedule and easy to implement, thus making it more realistic and reflective of situations of the known production condition. Weibull shock models with increasing failure rates are also considered as seen in the simulation example and the modified inspection schedule is used to improve the reliability of the whole production cycle.

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References