Optimal Expansion Planning of Traction Substations for an Electrified Mass Rapid Transit System

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Abstract—This paper is to investigate the proper expansion planning of traction substations (TSS) for an electrified mass rapid transit (MRT) system. The motion equation of train sets is used to solve the mechanical power consumption at each time snapshot according to the operation timetable, the passenger ridership and various types of operation resistance. The mathematical models of power converters in traction substations for different operation modes have been derived. With all train sets operated along the main line, the AC/DC load flow analysis is performed to find power demand of all traction substations for annual system peak operation over the study period. The objective function is formulated by considering both the voltage drop of train sets and investment cost of traction substations as the equivalent cost of all feasible states of each year. By performing the dynamic programming (DP) to minimize the objection function, the expansion planning of traction substations to achieve the minimum overall cost has been obtained by identifying the optimal capacity and locations of new traction substations to be committed at each year.

Index Terms—AC/DC Load Flow, Mass Rapid Transit (MRT), Dynamic Programming (DP), Traction Substation (TSS), 12 Pulse Uncontrolled Rectifier

I. INTRODUCTION

To solve the problems of traffic jam, air pollution, etc., in Kaohsiung metropolitan area, a Mass Rapid Transit System (KMRT) has been designed and constructed to provide an efficient transportation system with short headway, high ridership and less electric propulsion energy consumption. To maintain the high service reliability, the traction power network of MRT systems is always designed with fully backup capability to prevent the electricity outage due to the failure of any network component. It is the conventional practice to plan and install the traction substations (TSS) at the beginning with the capacity to cover the peak power loading of whole MRT system for the target year, which is maybe 30 years later. With relatively low ridership during the initial operation years of an MRT system and then gradually increased to the full operation capacity in the target year, the traction substations are lightly loaded over the life cycle.

For a typical MRT system, the electricity charge by utilities may contribute more than 20% of the total operation cost [1]. The enhancement of power system operation efficiency has become an important issue for the design of an MRT project. To improve the cost effectiveness of traction substations, the capacity expansion of traction substations has to be incorporated with the growth of ridership while subjects to meet the service reliability and performance index. Because the power demand of MRT propulsion system is increased with the growth of annual ridership, the expansion planning of traction substations is simulated as a dynamic programming (DP) problem [2] in this paper. The train performance simulation is executed to find the power demand and the location of each train set on the main line according to the mechanical traction power required to meet the timetable of train operation with various loading factors between stations. Each train set is then represented as a load bus and the DC network is updated according to the train locations at each time snapshot. To ensure the power service reliability, the system voltage profile and power demand of all traction substations are derived by AC/DC load flow analysis [3-6] for the N-1 reliability criterion. The unit commitment of traction substations is then determined to achieve the minimization of the investment cost of traction substations without violating the voltage drop of train sets and rated capacity of TSS for the study years. The flowchart of proper expansion planning of traction substations is illustrated in Fig. 1.

II. SYSTEM DESCRIPTION

In this paper, the voltage profile of an MRT system and the power demand of all train sets are simulated to solve the proper expansion planning of traction substations. With the annual growth of passengers, the headways of train operation are reduced accordingly to comply with the service performance index. The optimal capacity and locations of new traction substations to be committed to meet the system operation for each year are determined to achieve the best cost effectiveness of traction substation expansion. To solve the voltage levels of train sets to check the voltage limit violation and to calculate the power demand of all traction substations accurately, the mathematical modeling of traction substations, train borne converters with variable voltage and variable frequency (VVF) controllers and induction motors are included in the simulation of load flow analysis.
A. AC/DC power system of Orange line of Kaohsiung MRT

To illustrate the effectiveness of proposed expansion strategy of traction substations, the Kaohsiung Mass Rapid Transit (KMRT) system in Taiwan is selected for computer simulation. It consists of Orange line and Red line with track lengths of 14.1 km and 28.3 km respectively to serve the Kaohsiung metropolitan area. Fig. 2 shows the diagram of the traction power system of Orange line of KMRT system. One 63 MVA bulk power substations served by Taiwan Power Company (TPC) with double 161 KV circuits is used to provide the traction propulsion power for the train sets. Seven traction substations with capacity of 3 MVA each are installed for the initial stage of system operation at year 2006 and another six TSS are considered to be committed in the future as shown by the dash boxes to cover the system peak operation for the target year 2035.

B. Power system of propulsion traction

Fig. 3 shows the circuit diagram of traction substations and train sets. In each traction substation, a transformer is applied to step down the voltage level from 22 kV to 589 V and a 12 pulses rectifier is used to perform the AC/DC conversion to provide the 750 Vdc traction power source for train operation. A variable voltage and variable frequency (VVVF) inverter is installed in each train set to achieve adaptive control of frequency of ac voltage for the propulsion induction motors according to the speed profile between stations. The voltage magnitude is adjusted so that the proper torque can be generated by the propulsion system according to the ridership of the train set. When the train set decelerates during approaching the next station, the induction motors are operated with braking regeneration by converting the kinetic energy to electricity for energy conservation and provides the braking power required at the same time.

C. Mathematical models of traction substations

The traction substation of an MRT system is consisted of parallel uncontrolled rectifiers with an inverter as shown in Fig. 3. The 12 pulses traction substation is formed by two sets of 6 pulse diode rectifier bridge with 30° phase difference to provide the 750 Vdc power source to the third rail for train set operation. With the uncontrolled operation mode, the V-I characteristics of the rectifier is determined by its commutating impedance. The mathematic models for different operation modes of the rectifier and transformer set have been derived by authors [7]. The voltage and current at the dc side of rectifiers are represented as function of commutation angles and pre-ignition angles in equation (1), (2).
\[ E_a = \frac{6\sqrt{3}V_s}{\pi} \left[ \frac{\sqrt{2}(3+2\sqrt{3})}{7} \sin\left(\alpha_e - \frac{\pi}{6}\right) - \sin\left(\alpha_e + \mu + \frac{\pi}{6}\right) \right] \]

\[ I_a = \frac{\sqrt{6}V_s}{X_s} \left[ \sin\left(\alpha_e + \mu - \frac{\pi}{6}\right) - \sin\left(\alpha_e + \mu - \frac{\pi}{3}\right) \right] \]

The real power and reactive power at the ac side of the rectifier is derived as equation (3), (4).

\[ P_r = \frac{12\sqrt{3}V_s^2}{\pi X_s} \left[ \sin\left(\alpha_e + \mu + \frac{\pi}{6}\right) \Phi_{s1}(\mu) + \sin\left(\alpha_e + \mu - \frac{\pi}{3}\right) \Phi_{s2}(\mu) \right] \]

\[ Q_r = \frac{12\sqrt{3}V_s^2}{\pi X_s} \left[ \sin\left(\alpha_e + \mu + \frac{\pi}{6}\right) \Psi_{s1}(\mu) - \sin\left(\alpha_e + \mu - \frac{\pi}{3}\right) \Psi_{s2}(\mu) \right] \]

Here, \( \mu \) is the interphase commutation angle and \( \alpha_i \) is the postignition angle.

**D. Train control**

To achieve high transportation capability of MRT systems with rather short distance between stations, the train sets are operated with quick acceleration for start up propulsion and fast deceleration for braking stop. The automatic train operation (ATO) system is applied to control the magnitude and frequency of service voltage for induction machines so that proper propulsion traction effort can be provided and the time table of train operation can be maintained. For KMRT system, the operation modes of PWM, Quasi-Six steps and six steps are used to generate the trigger signal for the inverters for the speed range of 0-22 km/hr, 22-40 km/hr and 40-80 km/hr respectively [8]. When the train set leaves the station for acceleration, the constant torque mode of the propulsion system is applied for the train speed from 0 to 40 km/hr in Fig.4. After the train set has been accelerated with speed over 40 km/hr, the inverter supplies the induction motors with constant voltage for the operation modes of constant power and constant slip. To provide the same traction effort for the acceleration required, the boundary speed between different operation modes is adjusted with the third rail voltage according to the relationship of traction effort and speed at different voltage level [4]. When the train set approaches the next station for stop, the constant power regeneration mode is used for the speed above 56 km/hr. By this way, the induction motors can be prevented from over loading and sufficient braking effort can be obtained by adjusting the mechanical braking power. Once the speed has been reduced below 56 km/hr, the constant torque mode of inductor motors is applied to achieve full electric braking. When the speed has been reduce to be less than 5 km/hr, the electric brake is cut off and the mechanical braking is applied for the final stage of train stop.

**E. Propulsion power consumption of MRT train sets**

To solve the power consumption of train sets along the main line by performing the train performance simulation, the time table and speed profile of train operation between two substations is used to define the acceleration, coasting, deceleration and dwelling for each time snapshot of train operation. According to the ridership and traction characteristics of propulsion system, the power consumption and braking regeneration at each time snapshot is then derived by considering the motion equation of the train set in equation (5) [9].

\[ F_A = M_e a + R \]

Where \( F_A, M_e, a \) and \( R \) represent the traction effort, train mass, desired acceleration and operation resistance respectively. Fig. 5 shows the power and speed profile of a train set operated between two stations. At first, the train set starts up and the power consumption is increased during the acceleration stage. After train speed has reached the specified value after start up acceleration, the coasting mode is used to achieve operation efficiency by turning off the motoring power. With the braking regeneration, the kinetic energy has been converted to electricity energy by induction generation when the train set approaches the next station to make the stop.
F. Headway of train operation

With the increase of ridership over the study years, the headways of train operation have to be adjusted to meet the requirement of service criterion. In this paper, the annual ridership of orange line of KMRT system has been forecasted, which will be used to determine the headways for peak operation in Fig. 6. Based on the ridership and proposed headway, the voltage drop of train sets and investment cost of traction substations are integrated to form the equivalent cost of all feasible states for each stage or study year. The DP is then used to solve the optimal capacity and locations of new traction substations to be committed at each year to achieve the proper expansion planning of traction substations for the study MRT system.

G. Load sharing

The traction substations are considered to be installed at all train stations. For each traction substation, the ac power source from utility system is converted to 750Vdc by a 12-pulse rectifier for the 3rd rails. The traction power supply network is designed with the N-1 criterion so that the outage of a single traction substation won’t create any impact to the normal operation of train sets. The AC/DC load flow analysis is performed to find the power loading of all traction substations according to the train set operation. To simply the simulation process, the power loading of traction substations of all feasible states will be calculated by load sharing analysis [10] for the outage of any one TSS. Here, the resistance of rectifier transformer; \( R_{n,n+1} \) is represented loop resistance between traction substations \( n \) and \( n+1 \); \( V_{dn} \) is the source voltage at traction substation. The equivalent resistance \( R_{thn} \) is represented as eqn.(6)

\[
R_{thn} = R_{n,n+1} + \left( \frac{R_{dn}}{I_n} \right)
\]

By assuming the same source voltage levels for all TSS, i.e. \( V_{d1} = V_{d2} = \ldots = V_{dn} \), the current loading to be picked up by the other healthy traction substations are solved.

\[
I_k = \frac{R_{n+1}}{R_{n+1} + \Omega} \times I_n
\]

\[
I_k = \left( \prod_{i=1}^{n} \frac{R_{n+i}}{R_{n+i} + \Omega} \right) \times \frac{I_n}{R_{n+1} + \Omega} \times I_n
\]

Because the segment loop resistance \( R_{n,n+1} \) is much larger than the internal resistance of traction substation \( r_n \), the load transfer of TSS1 to the other traction substations has been solved as shown in Table 1. It is found that 84%, 12%, 2.73% and 0.52% of power loading of TSS1 have been picked up by TSS2, TSS3, TSS4 and TSS5 respectively.

III. DYNAMIC PROGRAMMING

The dynamic programming is often used to solve the unit commitment problem by reducing the computational effort in finding the optimal trajectories. In this paper, the DP has been used to solve the proper expansion planning of traction substations by considering the cost of traction substation investment and voltage drop of train sets. For the operation of Orange line of KMRT over the study period from 2006 to 2035, all the feasible states or traction transformer capacities of each stage are simulated. The cost of the feasible state \( Xi \) at stage \( i \) is determined as the summation of investment cost of traction substation \( CP \), and equivalent voltage drop cost \( CV \) to serve the system power demand in (9). The MATLAB is used to solve the DP routine with backward process to find the optimal expansion planning of traction substation capacity over the study period.

Objective function

\[
F^*(i, X_i) = \min_{X_{i+1}} \{ CP(i, X_i) + CV(i, X_i) + F^*(i+1, X_{i+1}) \}
\]

Subject to

1. \( V_{\text{min}} \leq V \leq V_{\text{max}} \)

2. \( P_{\text{TSS, cap}} \geq P_{\text{TSS, load}} \)

Equation (10) and (11) represent the voltage constraint of train sets and the capacity constraint of traction substations for the outage of any one TSS.
A. Objective function

To determine the proper expansion planning of traction substations for an MRT system, the objective function is formulated by including the cost of traction substation investment and voltage drop of train sets.

1. Energy consumption cost

The annual total energy loss of each feasible state can be solved by AC/DC load flow analysis according to the total power demand of whole MRT network. For instance, the state with \( j \) units of traction transformers at year \( t \) is calculated as

\[
CEL_j = EL(hw) \times NH \times W(hw) + EL(hw)_{op} \times NH_{op} \times W(hw)_{op}
\]

where \( hw, EL(hw), NH, \) and \( W(hw) \) represent the headway, total energy loss, total peak hours and energy charge rate respectively for peak period. \( EL(hw)_{op}, NH_{op}, \) and \( W(hw)_{op} \) are the total energy loss, total off peak hours and energy charge rate respectively for off peak period.

2. Capital investment cost

To simplify the capacity planning of traction substations, the expected life cycles of transformers and auxiliary equipments such as breaker, protection relays, etc. are considered. The annual operation and maintenance cost of traction transformers has been converted as the present value and incorporated with the procurement cost \( I \) to form the equivalent investment cost of traction transformers and auxiliary facility. The procurement cost of the 3-MVA traction transformers is then linearly depreciated over the lifetime of 30 years [11]. The capital investment cost of state with \( j \) units of traction transformers for year \( t \) is expressed as

\[
CINV_j = \begin{cases} 
1 \times 7 , & t = 1 \\
\frac{1}{30} \times (30 - k) \times j , & t = k \\
0 , & t \neq k 
\end{cases}
\]

The investment cost of traction substations with the energy loss cost are considered in the objective function by assuming flat inflation rate and interest rate over the study years. The equivalent present worth of the investment cost of traction substations is written

\[
CP = \sum_{j=0}^{\infty} (CEL_j + CINV_j) \times \left( \frac{1 + e}{1+i} \right)^t
\]

where \( i \) and \( e \) represent the inflation and interest rate respectively.

3. Voltage drop cost

With the outage of traction substations, the third rail voltage may drop to be less than 630V and the train set has to be operated with reduced performance to prevent the overload of traction substations. To ensure the power service reliability for the source outage of traction substations, the optimal capacity and locations of new traction substations to be committed at each year are determined to prevent the voltage drop violating of train sets. The AC/DC load flow analysis has been executed to solve the voltage drop of train sets according to the power demand of all train sets at each time snapshot. In this paper, the equivalent voltage drop of train sets cost \( C_V \) is expressed as equation (15).

\[
C_V = \begin{cases} 
0 , & V_{tid} \geq 630 \\
K \times \left( \frac{750 - V_{tid} - 0.8}{750} \right) , & 600 < V_{tid} \leq 630 \\
\infty , & V_{tid} < 600
\end{cases}
\]

where \( K \) and 750 represent the constant coefficient and the rated of third rail voltage. \( V_{tid} \) represent the minimum third rail voltage for year \( t \).

IV. OPTIMAL EXPANSION PLANNING OF TRACTION SUBSTATION CAPACITY

With the ridership forecast and operation timetable of the MRT network, the annual peak power demand for train set propulsion and system voltage drop are derived by executing load flow analysis. The backward dynamic programming is then used to solve the optimal unit commitment strategy of traction substations. To maintain the service reliability, the proper capacity reserve of traction substations is included in the computer simulation so that the performance level of all train sets will not be derated with the outage of any one TSS. After executing the dynamic programming, the proper expansion planning of traction substations for KMRT has been solved as shown in Fig. 8. It is found that the system peak power demand is increased with the annual growth of passenger ridership. To cover the peak propulsion traction power demand of MRT operation for each study year, the traction substations have to be committed at O11, O4, O13, O6, O8 and O9 for year 2012, 2014, 2018, 2023, 2027 and 2034 respectively.

![Fig. 8 Expansion planning of traction substations with system peak power demand.](image-url)
Fig. 9 Equivalent present value of investment cost of traction substations for proposed method.

Fig. 10 shows the loading factors and power loss of traction transformers for KMRT over the study period. With the original design of traction substations, the average loading factor of traction transformers is 60% and the average power loss is 147 kW. With the proposed traction substations planning strategy, the average loading factor has been improved to be 77% and the average power loss of traction transformers has been reduced to be 139 kW.

Fig. 11 Enhance meet to voltage drop of train sets by the proposed traction substations planning.

With the increase of ridership over the study period, the power demand and voltage drop of train set will be varied with the operating headways. Fig. 11 shows the voltage variation of train set before and after the installation of traction substations by the proposed method. It is found that the minimum voltage drop of train set is less than 600V for year 2012, 2014, 2018, 2023, 2027 and 2033. To ensure the power service reliability, the optimal capacity and locations of new traction substations to be committed at each year are determined without violating the voltage drop of train sets. It is found that the minimum voltage drop of train set has been improved effectively by the proposed method.

By executing the AC/DC load flow analysis for each time snapshot, the minimum voltage drop of train set can be identified. Fig. 12 shows the voltage variation of train set before and after the installation of proposed traction substations for year 2014. It is found that the minimum voltage drop of train set of 640V occurs at the 43rd snapshot, which is larger than the 630V constraint.

V. CONCLUSION

In this paper, the unit commitment of traction substations to serve the propulsion power demand of an MRT system in Kaohsiung Taiwan has been proposed. The dynamic programming has been applied to derive the proper expansion planning of traction substations by minimizing investment cost of traction substation without violation of voltage drop constraint. According to the ridership forecast and operation timetable of the KMRT network, the propulsion power demand of train sets and system voltage drop are derived by AC/DC load flow analysis. The objective function is formulated by considering both the voltage drop of train sets and investment cost of traction substations for all feasible states of each study year. By performing the dynamic programming, the optimal unit commitment strategy of traction substations has been solved to achieve best cost effectiveness of expansion planning of substations. With the proposed capacity expansion planning, the loading factor of main transformers has been improved and the power loss has been dramatically reduced.

VI. REFERENCES


VII. BIOGRAPHIES

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