HILBERT AND WAVELET ANALYSES OF UNSTEADY, THREE-DIMENSIONAL CHARACTERISTICS OF VORTEX SHEDDING FROM A CIRCULAR CYLINDER

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ABSTRACT

A comparison of of Hilbert and Wavelet analyses was made with the signals of the MEMS sensors situated spanwisely on a circular cylinder. The experiments were carried out at Reynolds numbers of 10⁵. The results of the Hilbert analysis clearly indicate that the events of vortex dislocation might be occurred in a localized spanwise region or over the entire cylinder for a time length up to several vortex shedding periods. Temporal variations of spanwise phase difference of vortex shedding reduced by the Hilbert analysis were noted in good agreement with those obtained previously by the Wavelet analysis. On the other hand, temporal variations of the Wavelet results appear to be more smoothed than those of the Hilbert results. This is attributed to the nature of windowed smoothing of the Wavelet transformation. The results obtained in this study are relevant to exploring the unsteady, three-dimensional behavior of vortex shedding.

I. INTRODUCTION

In recent years, a growing interest of applying Wavelet and Hilbert transformations to studying fluid flow phenomena has been noticed. Evidence reported in the literature indicates that these techniques could successfully assist in unveiling the detailed features of flow motions at a localized time. Farge [1] conducted a review on the theoretical studies of turbulence with the Wavelet transformation technique. Apart from the theoretical works, there are a number of experimental studies adopted the approaches of Wavelet and Hilbert transformations for data analysis. Li and Zhou [2] employed a two-dimensional discrete Wavelet transformation method to decompose the turbulent signals measured in the near wake region. The results indicated the contributions of different Wavelet components to the Reynolds stresses and vorticity. Buresti et al. [3] analyzed the hot-wire velocity signals obtained in the near wake region of a finite triangular cylinder with the Wavelet and Hilbert transformation techniques. The Wavelet transformation employed served to single out the components of interest, subsequently the Hilbert analysis was applied to extract the information contained in the components. Similarly, Zhang et al. [4] employed Wavelet analysis to filter out the noise components embedded in the signals of the vortex flowmeters. They claimed that the technique could effectively extend the lower measurement limit of the vortex flowmeters. Kareem and Kijewski [5] conducted a time-frequency analysis of wind effects on structures, in which they proposed a correlation method to extract the information from the Wavelet spectral results. In the correlation analysis, the coherence and bicoherence functions were introduced. Meaningful results were reduced from the filtered Wavelet coherence and bicoherence maps. Dunyak et al. [6] suggested a data reduction algorithm incorporating the technique of Wavelet transformation to identify the gust events from the wind data measured.

The present authors (JJM and CCH) conducted a Hilbert transformation analysis of the hot-film signals obtained in a water pipe flow. [7] In the experiment, a hot-film probe was introduced into the pipe flow to gather the information of vortex shedding produced by a vortex shedder. The Hilbert analysis provided the information of the instantaneous vortex shedding frequency subjected to the incoming flow velocity varying periodically in time. Therefore, by correlating the temporal variations of the instantaneous vortex shedding frequency with the incoming flow velocity, the results enabled one to identify the quasi-steady and unsteady regimes in reference to the ratio of the time-mean shedding frequency versus the characteristic frequency of the unsteady incoming flow.

Recently, the present authors (JJM and JKT) reported the findings of the three-dimensionality of vortex shedding based on the Wavelet analysis of the MEMS sensors signals obtained on the surface of a circular cylinder. [8] The instantaneous vortex shedding frequency of the MEMS sensor signals measured was identified from the Wavelet transformation analysis. Subsequently, the phases corresponding to the instantaneous vortex shedding frequencies at different spanwise locations of the MEMS sensors were reduced and compared to unveil the unsteady, three-dimensional flow behavior.

In addition, Miau et al. [9] employed the technique of Wavelet transformation to analyze the MEMS sensor signals obtained on the surface of a circular cylinder, in the neighborhood of flow separation. The percentage of time during which the vortex shedding frequency could be identified from the Wavelet analysis was obtained. The result is informative in providing a quantitative description on the unsteady behavior of flow separation. Wu et al. [10] conducted the Wavelet analysis of the hot-wire signals obtained in the wake behind a normal plate. By comparing the instantaneous phases of vortex shedding detected by two hot wires separated by a spanwise distance, in the free stream of the near wake region, they...
identified a trend that large spanwise phase difference, i.e., strong three-dimensionality, was likely occurred when the amplitude of velocity fluctuations measured was severely modulated. Based on the experimental findings, they further addressed that the formation of separation vortex in the near wake region can be categorized into two distinct situations, one of which is called Mode L, corresponding to the appearance of long vortex formation length, for which the spanwise phase differences of the two hot-wire signals measured appear comparatively small; and the other is called Mode S, which corresponds to the opposite situation.

Above reports enlighten the merit of applying Wavelet and Hilbert analyses to extract the information of the signals at a localized time. Particularly, the recent works of the present authors [8-10] showed that the phase information of the Wavelet results were valuable to explore the unsteady, three-dimensional behavior of the vortex shedding phenomenon.

In applying the Wavelet analysis in the previous works, [8-10] a question raised was concerning the issue that in reality the fluctuating energy of vortex shedding could not be fully represented by a single component of the maximum modulus identified in the Wavelet spectrum. Whether the findings obtained from the Wavelet analysis could adequately represent the flow phenomenon of interest or not deserves a careful look. On the other hand, the present authors noticed that the Empirical Mode Decomposition (EMD) procedure developed by Huang et al. [11] could also be applied to the measured signal. This procedure decomposed the signal into mono-components, one of which was subsequently identified as associated with vortex shedding. Physically speaking, the component would be more adequate to represent the total fluctuating motion of vortex shedding than that obtained from the Wavelet analysis. Note that the mono-component resulted from the EMD procedure can be analyzed by the Hilbert transformation directly. [11] Therefore, it is of interest in this study to compare the results of these two approaches, and examine the quality of the data reduced and the physical findings obtained.

II. EXPERIMENTAL METHOD

The present experimental setup was the same as that described in Tu et al. [8] However, for the completeness of this paper, the key features of the experimental method are described below. For more details, one may refer to Tu et al. [8]

Experiments were carried out in a low-speed wind tunnel, whose test section was 150 mm by 160 mm in cross section. Shown in Fig. 1 is a sketch of the present flow configuration that the circular cylinder spanned between two side walls in the test section. The diameter of the cylinder is 32 mm, denoted as D. Therefore, the aspect ratio of the cylinder is 5. Also shown in the figure is the coordinate system employed. $x$ denotes the axis along the streamwise direction, where $x=0$ is located at the center of the circular cylinder. $y$ and $z$ denote the axes along the vertical and spanwise directions, respectively.

Figure 2 presents a schematic drawing of the circular cylinder employed for the present experiment, on which five arrays of MEMS sensors were situated. The MEMS sensors were made of platinum 0.1 μm in thickness, deposited on a polyimide substrate as the flexible skin. The thickness of the polyimide substrate was less than 10 μm. Thus, the presence of the MEMS sensors would cause negligible impact on surface smoothness, aerodynamically speaking.

As indicated in the figure, only 14 sensors named S1 to S14 were selected for experiment. The sensors were positioned symmetrically with respect to the center of the span of the cylinder, denoted as $z=0$. Specifically, the sensors are at the spanwise locations: $z=±0.1D$, $±0.3D$, $±0.75D$, $±1.1D$, $±1.5D$, $±1.9D$ and $±2.3D$, respectively. It was further noted that the sensors S1 and S14 were actually immersed in the boundary layers on the two side walls, respectively.

The present Reynolds number, $Re$, is defined based on $U_0$ and $D$, which fall in the range of 10$^4$. $U_0$ denotes the reference velocity measured at the inlet of the test section.

The MEMS sensors were situated at the circumferential position $θ=75^\circ$, where $θ=0$ denotes the forward stagnation point on the cylinder. According to Miau et al. [9] whose flow configuration was identical to the present one, the on-set of flow separation from the circular cylinder was found to take place in the region of $θ=80^\circ-85^\circ$. Thus, the present MEMS sensors were immersed in the boundary layer upstream of separation.

To eliminate the drift component in the MEMS signals, which could be due to heat conduction between the substrate and the sensors under the unsteady flow conditions, a high-pass filter was incorporated in the amplifier circuit. Therefore, the resultant signals seen in the present paper contain no DC component.

III. INTRODUCTION OF HILBERT AND WAVELET TRANSFORMATIONS

Mathematically, Hilbert Transformation of a real-value signal $x(t)$ is given below. [12]

$$x_h(t) = \mathcal{H}[x(t)] = \int_{-\infty}^{\infty} \frac{x(t')}{\pi(t-t')} dt'$$

(1)

Let $z(t)=x(t)+i\dot{x}(t)$, which is an analytic complex function. $z(t)$ can also be written as $z(t) = A(t)\cdot e^{i\theta(t)}$, where $A(t)$ is called the envelope of $x(t)$, and $\theta(t)$ is called the instantaneous phase of $x(t)$.

$$\theta (t) = \tan^{-1} \left[ \frac{\dot{x}(t)}{x(t)} \right]$$

(2)
By differentiating \( \theta (t) \) with respect to time, the instantaneous frequency, \( f(t) \), of \( x(t) \) is obtained.

\[
f(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}
\]  

(3)

Note that a priori of applying Hilbert transformation to a signal is that the signal should contain no dc component.11 Along this consideration, Huang et al. [11] proposed the Empirical Mode Decomposition (EMD) procedure to decompose a continuous signal into a set of mono-components, each of which is ready for Hilbert transformation.

On the other hand, Wavelet transformation of a signal trace \( x(t) \) with respect to a mother wavelet function \( \psi(t) \) is defined by a convolution integral: [13]

\[
W(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi^{*} \left( \frac{t-b}{a} \right) dt ,
\]

(4)

where \( a \) and \( b \) are called the scale and translation, respectively, and \( \psi^{*} \) is the complex conjugate of \( \psi \). In this study, \( \psi \) is given as a continuous Morlet wavelet function.

\[
\psi(T) = e^{ik_{\psi} T} e^{-T^{2}/2} ,
\]

(5)

where the parameter \( k_{\psi} \) was taken to be 6.0 to satisfy the admissibility condition. [14] With \( \psi \) given in (5), one can see that the Wavelet transformation in (4) is like a finite time Fourier transformation.

According to Torrence and Compo, [14] the wavelet function \( W(a, b) \) in (4) can be analytically transformed into \( W(f, t) \), where \( f \) and \( t \) are the frequency and time, respectively, associated with the signals analyzed. As a result, \( W(f, t) \) can be referred as a time-frequency spectrum, which provides the spectral information of the signal at a localized time. For the present Wavelet analysis, the vortex-shedding frequency, \( f_{s} \), at a given time was identified as the one corresponding to a local maximum of the wavelet coefficients.

Further, the phase \( \theta \) of \( f_{s} \) at a time \( t \) can be defined below.

\[
\theta(f_{s}, t) = \tan^{-1} \frac{R(W(f_{s}, t))}{I(W(f_{s}, t))} ,
\]

(6)

where \( R \) and \( I \) denote the real and imaginary parts of \( W(f_{s}, t) \), respectively.

For the present application, the Hilbert analysis provides the phase information of vortex shedding from (2), then the frequency is reduced as the time derivative of the phase function; on the other hand, the Wavelet analysis gives a Wavelet spectral function, from which the vortex shedding frequency can be identified, then the corresponding phase is reduced from the relation of (6).

VI. RESULTS AND DISCUSSION

An example of applying the EMD procedure to the measured signal prior to Hilbert analysis is given in Fig. 3. Figure 3a presents a raw MEMS signal trace together with the EMD mono-component of the sensor S7 over one second, at \( Z=0.1D, \) for \( Re=2.7 \times 10^{4} \). Moreover, two segments of the EMD mono-component within the time intervals of \( t=0.1-0.3 \) and 0.8-1s are highlighted in Fig. 3b, which provide a detail view of the events whose amplitudes are strongly modulated. In the figure, the segments of the corresponding original trace are also included for comparison. As seen in these plots, the mono-component reduced from the EMD procedure, which represents the vortex shedding frequency component of interest, evidently dominates the original trace. Further noted, around \( t=0.18s \), an irregular undulation is seen in the original trace, but not in the mono-component; around \( t=0.85s \), both of the original trace and the mono-component show irregular undulations over a time length about three vortex shedding periods.

Two more examples of comparing the original traces and the EMD mono-components of the sensors S3 and S10 are given in Figs. 4 and 5, respectively. Note that the traces of S7, S3, and S10 shown in the figures were sampled simultaneously. Later, attention will be paid to the instantaneous phase differences between the mono-components of the sensors during these time intervals.

According to the results of the EMD/Hilbert analysis of the signals of S3-S12 within \( t=0-1s \), Fig. 6 shows the temporal variations of spanwise phase differences of vortex shedding, in reference to the phase of S3. Interesting features observed from this plot are described below. Firstly, the appearances of sudden jumps of 2N \( \pi \) (N denoting integer) about \( t=0.1, 0.2 \) and 0.85s are noted very remarkable, which can be further identified as the occurrences of vortex dislocation, the phenomenon pointed out in a number of previous studies, for instance, Gerrard [15] and Williamson. [16] In the earlier studies, the phenomenon of vortex dislocation was clearly observed at low Reynolds numbers by the conventional flow visualization techniques. But, for the Reynolds numbers of \( 10^{5} \), visualizing the vortex dislocation phenomenon with the conventional techniques would be rather difficult. Secondly, it is noted from the figure that the vortex dislocation might take place in some localized regions, for instance, \( t=0.1s \) an event took place in the vicinity of S11, and \( t=0.2s \) the other event took place over a spanwise region of S10-S12. On the other hand, in the neighborhood of \( t=0.85s \) sudden phase jump prevailed over all the locations of the sensors, inferring that the orderly vortex shedding structures along the cylinder were bursting into a chaotic manner entirely. Such a bursting event could last over several vortex shedding periods, before returning to the orderly appearance. Note that the events mentioned above are occurred within the time intervals that the fluctuating amplitudes seen in Figs. 4 and 5 are strongly modulated. Similarly, Williamson pointed out that the phenomenon of vortex dislocation
took place at the time instants when the vortex shedding signals appeared strongly modulated.

Tu et al. [8] noticed that strong three dimensionality of vortex shedding was featured with pronounced variations of the instantaneous vortex shedding frequencies over the span of the cylinder. Therefore, the variance of the instantaneous vortex shedding frequencies reduced at different spanwise locations was proposed as an indication of the three-dimensionality. Referring to Tu et al. [8] and the present study, one can further argue that strong three-dimensionality of vortex shedding is often associated with the occurrences of vortex dislocation, where the signal amplitudes appear strongly modulated.

In addition to the results of the EMD/Hilbert analysis shown above, it was seen in the previous study [8] that the Wavelet analysis could perform satisfactorily to provide the localized time information of the unsteady, three-dimensional characteristics of vortex shedding. In order to examine the consistency of the results obtained by these two methods, Fig. 7 compares the temporal variations of phase difference of S7 to S3 reduced by the two methods, respectively. As seen, the results of Hilbert analysis contain more small scale fluctuations compared to those of Wavelet analysis, but the global trends of the two curves agree quite well. This comparison evidences that the findings of the two methods be consistent. Further, this clarifies a concern mentioned earlier that the results of Wavelet analysis, which literally speaking is associated with a single frequency component of the Wavelet transformation, can adequately describe the flow phenomenon. On the other hand, the Wavelet analysis data in the figure appear to be much more smoothened than that of the Hilbert analysis data. This is attributed to the present Wavelet transformation is in effect a window-smoothing Fourier transformation of a finite time. Thus, small-scale fluctuations could be smoothened out in the windowed averaging process.

Figure 8 further compares the results of the instantaneous vortex shedding frequency, $f_s$, obtained by the two methods for the S7 signal trace obtained at $Re=2.7 \times 10^4$ over one second. Similar to Fig. 7, the comparison reveals that the results of the two methods agree quite well as far as the global trend is concerned, but the results of the Hilbert analysis contain pronounced small-scale fluctuations which obscure the global trend to a great extent. The pronounced small-scale fluctuations resulted from the Hilbert analysis is mainly attributed to that the quantity of $f_s$ was reduced according to (3), which is a differentiation of phase with respect to time. As known in numerical analysis, [17] differentiation results in larger error or uncertainty compared to that of the original quantity.

V. CONCLUDING REMARKS

In this study, the EMD procedure together with Hilbert transformation was successfully applied to analyze the MEMS sensor signals obtained on the surface of a circular cylinder at Reynolds number of $10^4$. The major findings are that the events of strong three-dimensional vortex shedding were identified and attributed to the occurrences of vortex dislocation. These events might take place either in a localized spanwise region or over the entire cylinder. These events could last over several vortex shedding periods, before the vortex shedding structures returned to a more or less two-dimensional appearance.

The results of the Hilbert analysis obtained in this work are found in good agreement with those of the Wavelet analysis reported earlier. [8] The approach of the Hilbert analysis is dealing with the mono-component of the signal, which is referred to as a better representation of the entire vortex shedding component, compared to the single frequency component of the Wavelet analysis. Nevertheless, the comparison validates the physical findings inferred by the previous Wavelet data. [8]

Comparing the temporal variations of spanwise phase difference of vortex shedding reduced from the present Hilbert approach and the previous Wavelet approach reveals that the latter provides the data in a much more smoothened manner with respect to time compared to the former. This is attributed to the nature of windowed smoothing of the present Wavelet transformation. The instantaneous vortex shedding frequency reduced by the Hilbert analysis show pronounced fluctuations. This is noted due to that this quantity was reduced from a differentiation procedure.

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mono-component reduced from the EMD procedure within the time intervals of $t=0.1-0.3$ and $0.8-1s$, respectively.

Fig. 4 (a) and (b) Comparison between the raw signal trace of S3 and the mono-component reduced from the EMD procedure within the time intervals of $t=0.1-0.3$ and $0.8-1s$, respectively.

Fig. 5 (a) and (b) Comparison between the raw signal trace of S10 and the mono-component reduced from the EMD procedure within the time intervals of $t=0.1-0.3$ and $0.8-1s$, respectively.

Fig. 6 Temporal variations of spanwise phase differences of vortex shedding, with the phase of S3 as the reference.

Fig. 7 Comparison of the temporal variations of phase difference of S7 to S3 reduced by the EMD/Hilbert and Wavelet methods, respectively.

Fig. 8 Comparison of the results of the instantaneous frequency obtained by the Hilbert and Wavelet analyses for the S7 signal trace obtained at $Re=2.7 \times 10^4$ over one second.